

On measuring bank funding liquidity risk

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Presented at the Actuarial Society of South Africa's 2013 Convention
31 October–1 November 2013, Sandton Convention Centre

ABSTRACT

The standard framework to measure bank funding liquidity risk compares expected cumulative cash shortfalls over a particular time horizon against stock of available funding sources. This requires assigning cash-flows to future periods on financial products with uncertain cash-flow timing. There is lack of consensus on how to assign such cash-flows. So far, existing models employed to assign cash-flows give little credence to the distribution of time that financial products remain on a bank's book. Thus, little is known about the empirical run-off distribution of most bank financial products. Against this background, this study formulates a method to handle cash-flow timing uncertainty when measuring bank funding liquidity risk at a business unit level on a run-off basis. A state-dependent non-parametric survival model is employed to assign cash-flows to future time horizons. The resulting model is illustrated on a case study of a bank savings product.

KEYWORDS

Funding liquidity risk; cash-flow modelling; indeterminate maturity deposits; withdrawal rates; run-off rates; non-maturing assets and liabilities

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1. INTRODUCTION

The framework for quantitative measurement of bank funding liquidity risk is fairly standard. These measurements include: net cash capital position, maturity mismatch profile, liquidity coverage ratios and funding ratios. Calculating these measurements is straightforward if cash-flow timing is certain. However, some bank financial products have uncertain cash-flow timing. Such products are referred to as having ‘indeterminate maturity’. Equally, the term ‘indeterminate maturity’ can refer to a financial product whose cash-flow timing differs significantly from that specified on the product’s contract (Bardenhewer, 2007). Examples of bank assets with uncertain cash-flow timing include credit card accounts and overdraft accounts. For bank liabilities, examples include demand deposits and savings accounts. Products with uncertain cash-flow timing constitute a significant portion of a typical depository institution. For example, as at 30 June 2012, indeterminate maturity deposits on USA banks were approximately 57% of total assets, based on data from the Federal Deposit Insurance Corporation (Matz, 2013). Thus, it follows that an accurate understanding of the liquidity risk characteristics of indeterminate maturity products is of significant practical importance to bank funding liquidity risk measurement (Jarrow & van Deventer, 1998).

Banks face two fundamental problems in assessing funding liquidity risk:

- (1) Assigning cash flows to future time horizons on financial products with uncertain timing; and
- (2) Ascertaining the stable (core) and less stable (volatile) portion of financial products with uncertain cash-flow timing (Kalkbrener & Willing, 2004). Bank regulators face similar problems on setting and monitoring funding liquidity requirements such as the liquidity coverage ratio and net stable funding requirement (de Haan & van den End, 2013).

Concerns on the few quantitative models, in the literature, that handle the cash-flow timing problem of bank products are threefold:

- (1) The models infer a financial product’s run-off profile¹ through studying its volume time series. Nonetheless, a volume time series shows how a financial product’s balance position evolved over time, not necessarily the time those balance positions stayed with the bank. Thus, it seems reasonable to assume that modelling time that balance positions stay on a bank’s book better matches the objective of allocating cash-flows to future time horizons.
- (2) Quantitative models in the literature utilise parametric approaches, which restrict the run-off pattern. However, the literature is silent on determining non-parametric run-off patterns of financial products.
- (3) Although liquidity risk assessment is scenario-specific, existing models are often calibrated without duly considering the liquidity scenarios that prevailed in the past.

¹ A run-off profile draws a picture of how balances on a bank product reduce over time, ignoring future business.

This study contributes towards addressing these epistemological problems in funding liquidity risk assessment through developing a unique time-to-event model to handle cash-flow timing uncertainty. Financial product positions are the subject of study. Of interest for these subjects is the time they are retained on a bank's book. To date, empirical knowledge on run-off profiles for bank products with indeterminate maturities is limited to a few studies calibrated without considering the time positions stay on a bank's book. This limitation implies that many important questions remain unanswered. What form is the distribution of run-off profiles? How do run-off profiles vary across scenarios (stressed or 'normal' operating environment)? Are run-off rates proposed under Basel III reasonable? To what extent does stability of different funding sources differ from each other? This study attempts to demonstrate how these and related questions can be answered. A case study of a bank savings product is used to illustrate the model.

In this study, funding liquidity risk is defined as the risk that an economic agent fails to fund its net cumulative cash outflows over a specific period. Four features of this definition are worth noting. Firstly, it ignores the economic agent's solvency status. The reason is that funding liquidity problems occur regardless of solvency status. Moreover, it is questionable whether economic agents always first consider solvency before funding liquidity. Secondly, this study's funding liquidity risk definition disregards the cost of liquidity. Although unlikely, one can still face funding liquidity risk problems even if the cost of liquidity remains unaltered. For instance, funding liquidity risk problems can arise solely from increased onerousness of collateral requirements: i.e., lenders requiring collateral that is less risky and/or more easily convertible to cash. Thirdly, the definition disregards the element of obtaining funding with immediacy. This is because immediacy is considered as a market liquidity risk feature since it is more of a transactional property of markets where funding is sourced (Kyle, 1985). In this sense, immediacy is one way that market liquidity risk affects funding liquidity risk (Brunnermeier & Pedersen, 2009). Fourthly, the definition is consistent with the approach by Drehmann and Nikolaou (2013) of distinguishing between funding liquidity and funding liquidity risk. As pointed out by Drehmann and Nikolaou (2013), funding liquidity is a binary concept: an agent can either fund or is unable to fund its obligations. In contrast, funding liquidity risk is defined on a continuum (that is, on a probability space) and is forward looking. In conclusion, while this study's funding liquidity risk definition is somewhat general, it does provide a reasonable basis for measurement because it excludes subjective elements.

The paper proceeds as follows. Section 2 reviews the literature on bank funding liquidity risk measurement. Section 3 presents a survival model to estimate run-off profiles of bank financial products. Section 4 presents a case study on determining the run-off profile of a savings product. Finally, section 5 concludes.

2. LITERATURE REVIEW

Methods employed in the literature to estimate cash-flows emerging from an indeterminate maturity product can broadly be classified into four groups:

- (1) Time series methods;
- (2) Replicating portfolio methods;
- (3) Stochastic methods; and
- (4) Retention curve methods.

The same methods are used to determine the stable portion of an indeterminate maturity product, which is defined as the portion anticipated to remain on a bank's balance sheet for at least one year (Neu, 2007; Basel Committee on Banking Supervision, 2010). This section starts by reviewing each of the four methods. Lastly, the section motivates the use of state-dependent non-parametric survival methods as contributing toward addressing shortcomings noted in existing models.

2.1 Time Series Methods

Time series methods deduce an indeterminate maturity product's run-off profile from a statistical distribution of the account volume trajectory. Two classic papers that employ time series methods are Neu (2007) and Vento and La Ganga (2009). In these papers, the future account balance of an indeterminate maturity product is modelled by a log-linear time series regression where the dependent variable, a log-transformation of the account balance, is explained by the existing account balance (the intercept), time (trend) and a normally distributed error term. This regression equation is then used to infer the run-off, at a given level of confidence, considering the mean and volatility of the historical account balance.

2.2 Replicating Portfolio Methods

The replicating portfolio method constructs a portfolio deemed to closely resemble the indeterminate maturity product in terms of cash-flow properties such as the currency, and timing of cash-flows. Constituents of the replicating portfolio comprise of standard traded financial instruments, such as fixed income securities, money market instruments and standard swaps. Once constituents are selected, what remains is to determine their weight in the replicating portfolio. Cash-flows emerging from the replicating portfolio are then equated to those from the indeterminate maturity product.

Weights of the replicating portfolio's constituents are typically estimated in an optimisation framework. Most models specify what is optimal in terms of the difference between return on the replicating portfolio and interest rate on the indeterminate maturity product. Frauendorfer and Schürle (2007) determine portfolio weights by minimising the expected downside deviation of the spread between the yield on the replicating portfolio and on the indeterminate maturity product. In contrast, Maes and Timmermans (2005) use the standard deviation of the same spread. Other constraints placed on the optimisation problem include the following:

- (1) the weights of financial instruments in the replicating portfolio sum to one (e.g. Maes & Timmermans, 2005; Bardenhewer, 2007); and
- (2) weights of constituent financial instruments are non-negative. That is, short position holdings are not permitted.

While most studies apply static weights, Frauendorfer and Schürle (2007) use dynamic weights, meaning that the replicating portfolio weights change over time with changes in indeterminate maturity products' volume, interest rate and market interest rates.

In Bardenhewer (2007), indeterminate maturity product cash-flows are allocated using a three stage approach. Stage one involves projecting an indeterminate maturity product's account balance. Thereafter, the series is split into a deterministic trend and a random component, indicating deviation from the trend. The deterministic trend is modelled by a time series regression where the dependent variables are: the current indeterminate maturity product's volume (the intercept); time; and deviations of the indeterminate maturity product's interest rate from its historical average. This differs from Neu (2007) and Kalkbrener and Willing (2004) who exclude interest rates as an explanatory variable. At each projected future time, the trend volume is invested in a replicating portfolio. The remainder, i.e. the random component, is treated as the indeterminate maturity product's cash-flow realised within one month. Stage two involves determining the weights of replicating portfolio constituents that minimise volatility of the difference between return on the replicating portfolio and the interest rate on the indeterminate maturity product. Allocating cash-flows for the trend volume is deduced from the cash-flow profile of the replicating portfolio. Stage three involves moderating the run-off profile.

2.3 Stochastic Methods

A seminal paper applying stochastic methods to determine the term structure of liquidity is that by Kalkbrener and Willing (2004). They develop a model to value indeterminate maturity liabilities that accounts for liquidity risk and interest rate risk. In their model, liquidity risk is future uncertainty in the indeterminate maturity liability's account balance. To project future account balances, Kalkbrener and Willing (2004) assume that account balance increments are governed by a normal distribution. They also suggest a log-normal distribution as an alternative specification of the volume increments. Several paths of the account balance process are then obtained by simulation. For each account balance trajectory, the existing portfolio run-off is obtained from the minimum account balance in earlier periods along that path. Thus, for each simulated account balance path, there is a corresponding simulated run-off path. At each projection time, t , the run-off account balance is then obtained, at a given level of confidence, by considering the distribution of simulated values as at time t . For example, say the simulation procedure generates 10,000 run-off paths. Further, say the objective is to estimate the run-off account balance, at time t , likely to be available with at least 95% level of confidence. This is the 95th (lower) percentile of the 10,000 run-off values projected as at time t .

2.4 Retention Curve Method

Poorman and Stern (2012) pioneer the use of retention curves, which are functions showing the probability that an account is retained by a bank for a given period. Poorman and Stern (2012) fit the retention curve to a Weibull distribution. Leonard Matz (2013) develops retention curves by evaluating the percentage of original deposit still retained by a bank over time.

2.5 Motivation for using State-dependent Survival Approach

With the exception of the retention curve method, it appears that literature on quantitative models of how indeterminate maturity product cash-flows evolve are grounded on analysing the stock of an indeterminate maturity product's account volume. Yet a stock variable lacks information on the time that indeterminate maturity product positions are retained by a firm. It is thus questionable whether analysing a stock variable is consistent with the objective of estimating cash-flows emerging from the existing indeterminate maturity product's account balance. Instead, it seems more appropriate to utilise a model that analyses the time indeterminate maturity product positions stay with a bank.

In Kalkbrener and Willing (2004), Neu (2007) and Vento and La Ganga (2009), volume increments are modelled by either a normal or lognormal distribution. The resulting run-off profiles from these models are thus from a parametric model. There is a gap in literature on non-parametric model specification. We thus lack a good understanding on the empirical distribution of unrestricted run-off profiles.

The models by Kalkbrener and Willing (2004), Bardenhewer (2007), Neu (2007) and Vento and La Ganga (2009) have parameter values influenced by addition of positions on a bank's book. The impact of 'new positions' is to increase volatility of the stock variable, which is a key statistic influencing the outcome of models discussed in the literature. Thus, a better model, in terms of the objective of allocating future cash-flows, is one that is calibrated excluding 'new positions' but focuses on the indeterminate maturity product's run-off.

The method to optimise weights of assets in the replicating portfolio can result in cash-flow patterns that differ from the indeterminate maturity product's run-off. This explains why Bardenhewer (2007) moderates cash-flow patterns derived from the optimisation procedure. Such moderation indicates potential limitations of using replicating portfolio approaches in measuring funding liquidity risk.

The timing of cash-flows is exogenous to the replicating portfolio approach. That is, the model assumes that the replicating portfolio closely resembles the indeterminate maturity product with respect to cash-flow timing. What seems missing in the literature is a methodology to ascertain the extent to which the replicating portfolio of assets matches the cash-flow timing of the indeterminate maturity product. This study contributes toward filling this gap by producing a non-parametric run-off distribution. Unlike the replicating portfolio method, this study develops a model where cash-flow timing of indeterminate maturity products is determined endogenously.

To date, models that estimate cash-flows emerging from indeterminate maturity products are calibrated ignoring the liquidity scenarios that prevailed during the period data is collected. As a result, associating run-off profiles to particular liquidity scenarios becomes less clear. This study contributes towards addressing this gap by taking into account liquidity scenarios when calibrating run-off profiles.

The model developed in this paper is, in spirit, similar to recent papers describing retention curves by Matz (2013) and Poorman and Stern (2012). This study, however, differs from these recent papers on the following issues. Firstly, this study provides a theoretical conjecture to underpin the use of lifetime modelling in estimating run-off profiles. By so doing, this paper explicitly handles lifetime data problems, i.e. censoring and truncation. Secondly, this study develops an algorithm to determine the time origin of account balances, unlike Matz (2013) and Poorman and Stern (2012) who treat time origin only as the date an account is opened. Thirdly, this study develops state-dependent run-off profiles.

3. SURVIVAL MODEL FORMULATION OF FUNDING LIQUIDITY RISK

This section shows how bank data can be processed to infer the run-off profile on a class of business using survival methods. Here, a class of business comprises two, or more bank products. This section however interchanges ‘business class’ with ‘indeterminate maturity product’.

3.1. Estimating the Run-off Profile

The subject of this study is one-hundredth of a monetary unit (cent) held in an account. Let V_i denote the balance on account i . Suppose account i has a balance of R100.00, then $V_i = 10,000$. For each subject, the observation of interest is the time that the subject is on a bank’s book: denote this variable by T . In terms of cash-flow, specifying T depends on whether one is considering an asset or a liability. From the bank’s point of view, if a subject is a liability, e.g. a savings account, then exit of a subject from a bank’s position corresponds to a cash outflow. In contrast, if the subject is an asset, e.g. a credit card account, then exit from a bank’s position corresponds to a cash inflow.

It is assumed that the distribution of T for each subject is independent of other subjects. Admittedly, the subjects for this study are grouped into accounts and the survival times of subjects within an account tend to be correlated because the subjects share a common account holder. The effect of such intra-account correlations, however, is not to alter the ‘survival’ function estimates, but its variance (Williams, 1995; Williams, 2000). Although validity of the independence assumption is questionable for subjects belonging to a single account holder, it is reasonable when considering subjects derived from several account holders. In principle, the more diversified the number of account holders the more reasonable the assumption of independence.

Let V_t denote the total account balance on a class of indeterminate maturity business at time t . Similarly, let $V_{i,t}$ denote the balance on account i at time t . The relationship between the two variables, is shown in Eq. (1).

$$V_t = \sum_i V_{i,t} \quad (1)$$

As mentioned earlier, models in literature often calibrate run-off models on a time series study of V_t (see for example, Neu, 2007; Vento & La Ganga, 2009). Such an approach has the problem that V_t does not necessarily inform us about the time that a particular position remains on a bank's book, which is the most important aspect to consider if the objective is to allocate cash-flows to future time horizons.

To remedy the above problem, this study observes the time subjects under study stay on a bank's book. Thus, increments on an account are ignored as these are treated as subjects outside the study. To illustrate, consider the following example: Suppose a savings account has a balance of R100.00 at inception. This account thus contributes 10,000 subjects to the study. Suppose the account holder deposits R50.00 a week later. Clearly, this additional R50.00 is uninformative of the time that the 10,000 subjects under study have stayed on the bank's book. Put differently, the additional R50.00 is uninformative of the run-off as at $t=0$. In sum, what matters for this study are only decrements (withdrawals for liabilities and repayments for assets), not increments (deposits for liabilities and drawdowns for assets) since increments lack run-off information.

As a result of being interested only in decrements, there is need to define a monotonically decreasing function of the account balance, thereby enabling the study to employ survival methods as a tool in allocating cash-flows to future time horizons. This is done for account i in Eq. (2).

$$\tilde{V}_{i,t} := \min_{0 \leq s \leq t} V_{i,s} \quad (2)$$

In Eq. (2), $\tilde{V}_{i,t}$ is a function that tracks only decrements of subjects that existed at inception ($t=0$). While Eq. (2) is structurally identical to that in Kalkbrener and Willing (2004), they serve different purposes. Kalkbrener and Willing (2004) use Eq. (2) to determine a run-off path from a simulated path of the future account volume. In contrast, this study employs Eq. (2) to determine the observed run-off from past data of the account volume. In essence, while Kalkbrener and Willing (2004) derive the run-off after calibrating their model on account volume, this study derives the run-off on calibrating the model.

For simplicity, this study treats business classes independently of each other. Such an approach is similar to that taken by other studies (see, for example Kalkbrener and Willing, 2004; Neu, 2007; Vento & La Ganga, 2009). A consequence of treating business classes independently is that when studying the distribution of time on a bank's position for a business class, internal transfer of funds to other business class accounts is treated as a form of censoring.

Survival analysis can be employed to estimate the proportion, $P(t)$, of the volume of a business class whose time on a bank's position exceeds t without imposing any distribution on $P(t)$. Formally, $P(t)$ is defined as:

$$P(t) = \Pr(T \geq t) \quad (3)$$

To obtain $P(t)$, use is made of $\tilde{V}_{i,t}$. Consider a discrete time framework where $\tilde{V}_{i,t}$ decreases over the time step from time $t=0$ to time $t=1$. That is, run-off on account i is $\tilde{V}_{i,0}$ to $\tilde{V}_{i,1}$. The difference, $(\tilde{V}_{i,0} - \tilde{V}_{i,1})$, can be explained as resulting from either the subjects exiting a bank's book or censorship (e.g. account suspension). Thus, after each time step, we observe subjects exiting from a bank's book, censored observations and the subjects remaining on the bank's book. By way of analogy, the subjects exiting a bank's book are treated as 'deaths' in traditional lifetime studies. Account-level data on balances can thus be structured in a form required to perform traditional survival analysis (discrete and continuous time), thereby providing run-off patterns of financial products.

3.2 Implications of using Survival Models in Funding Liquidity Risk Measurement

While this study is grounded on applying survival models, such models were originally developed to model lifetime data, not cash-flow timing. As such, it is critical to question the suitability of survival model assumptions when applied to cash-flow modelling. This section evaluates the applicability of survival model assumptions in the context of cash-flow modelling when measuring bank funding liquidity risk.

What are the similarities in modelling cash-flows and lifetime? Key similarities are mentioned below. Firstly, both cash-flow modelling and lifetime modelling model the time that a subject remains in a particular state. Typically, in lifetime modelling, the subject can either be 'alive' or 'dead': in this case, the variable of interest is the time to 'death'. Similarly, a cash-flow position can either be on a bank's book or not on a bank's book: the variable of interest is thus the time a position on an indeterminate maturity product is retained on a bank's book. Secondly, both cash-flow and lifetime modelling are censored: that is, only partial information on the 'survival' time is known for some of the subjects under observation. Generally, censorship is endemic in an experimental design where 'survival' times are observed over a limited experiment period.

What are the key differences between modelling cash-flows and lives? In the context of measuring bank funding liquidity risk, there are two key differences. Firstly, cash-flow modelling is scenario dependent whereas standard lifetime modelling is typically modelled as scenario independent. In fact, the objective under cash-flow modelling is to obtain 'survival' distributions under different scenarios. This is irrespective of whether the survival analysis is extended to incorporate a regression model to estimate the relationship between covariates and 'survival' times. In contrast, lifetime modelling is concerned with the estimation of a single 'survival' time distribution, which is scenario independent. This remains the case even when the 'survival' analysis is extended to incorporate covariates. Secondly, the time origin to base 'survival' analysis of financial products is unclear unlike the case with traditional fields where 'survival' models have been applied. The section below proceeds to discuss

the approach taken to handle the time origin problem in the context of modelling cash-flow timing.

3.3 Determining the Time Origin for an Account’s Run-off Profile

Specifying the time origin is important in time-to-event modelling since, for a given survival dataset, the resulting survival curve varies with time origin definition (Sperrin & Buchan, 2013). Examples of time origin definitions in survival studies include: birth date; entry date to an experiment; start date of an experiment; and date an object of study attains a specific state. In most survival studies, defining time origin is straightforward and intuitive. Moreover, the time origin can easily be established for a subject under study. The suitable time origin on a bank account is, however, less clear because it is impossible to distinguish between monies in that account. This difficulty is handled in this section by utilising an algorithm to ascribe the time origin on ‘lives’ in an account at a specific point in time.

The time origin is ascertained when observing an object of study. By way of analogy, demographic studies define time origin at the time a census occurs. Put differently, the census date is the reference date whence one determines the time origin of subjects under study. In this study, the term ‘base date’ is analogous to ‘census date’. The base date can also be viewed as the experiment start date.

On a specific base date, b , let τ_i denote the time origin of ‘lives’ in account i . Further, let t^- denote the time just before time t ; and let $\mathcal{F}_{i,b}$ denote historic information about account i up to, and including, the base date. Given these notations, and assuming a constant liquidity scenario, the time origin is defined in Eq. (4).

$$\tau_i := t : \left\{ \underbrace{(V_{i,t} \leq V_{i,u}) \forall_{t \leq u \leq b}}_{\text{Monotonically decreasing}}, \underbrace{V_{i,t} > V_{i,t^-}}_{\text{Local maxima}} \mid \underbrace{\mathcal{F}_{i,b}}_{\text{Filtration of the account balance process}} \right\} \quad (4)$$

Putting Eq. (4) in words, the time origin on account i is time, t , corresponding to the furthest local maxima of the account balance history when moving back in time from the base date.

Fig. 1 presents an example of employing Eq. (4) to deduce the time origin on a savings account on a given base date. Specifically, Fig. 1 shows the savings account balance history of an account opened at time t_1 , with an initial deposit of V_1 . Suppose that time t_4 is the base date. A question then arises: On the base date, what is the appropriate time origin for the savings account: t_1 , t_2 , t_3 or t_4 ? At time t_4 , knowledge about the account’s run-off can be obtained from the previous account balance information. Therefore, setting t_4 as the time origin results in omitting available run-off data, thereby causing bias. Setting time t_3 as the time origin is inappropriate because additional run-off history can be extracted by going further back in time. Time t_2 is one potential time origin since this is the furthest time that provides the experiment

with the highest amount of subjects to study. Time t_1 is also a plausible time origin in the sense that one can determine a run-off from time t_1 . This is, however, at the expense of excluding the additional deposit of $(V_2 - V_1)$ at time t_2 . To summarise, t_2 and t_1 are equally sensible time origins. This study opts for t_2 as the time origin on the grounds that doing so maximises the number of studied subjects at a given base date. In conclusion, information on the run-off for an account can be inferred from observing the past trajectory if, and only if, backward movement from the base date is non-decreasing for a least one time unit. Otherwise, the time origin is the base date.

On a specific base date, the time origin varies across accounts. This is illustrated in Fig. 2, where N accounts exist on the base date. In Fig. 2, Account 3's time origin is the base date, while Account 1, 2 and N have different time origins before the base date. For each account, the time origin is the time from when run-off on that account starts being evaluated as at the base date. After the base date, the run-off profile on each of the N accounts is calculated using Eq. (2). Run-off data on the N accounts is then aggregated to draw a picture of the run-off profile on the class of business. The run-off profile's value date is the base date. This is analogous to associating results on a demographic study to a specific census period.

3.4 Merits of estimating a Run-off Profile using Estimates from Multiple Base Dates

As highlighted in section 3.3, a run-off profile on a class of business is estimated in respect to a specific base day. The run-off on a class of business can also be inferred from multiple run-off estimates derived from multiple base days. This is similar to

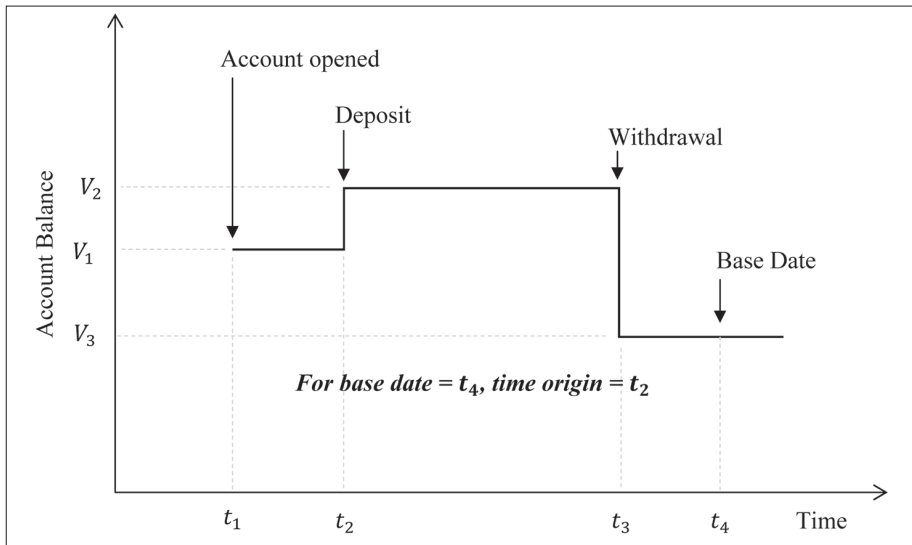


FIGURE 1 Time origin example on individual account

running an experiment multiple times and then using the distribution of experiment results for inference. Formally, let M be the total base days. For each $b(1 \leq b \leq M)$, let $\hat{S}_b(t)$ be the maximum likelihood estimator of $S(t)$ based on the run-off profile associated with base day b and let w_b be the weight for $\hat{S}_b(t)$. Then, for a given t , the mean estimate of $\hat{S}(t)$ is given by:

$$\bar{S}(t) = \sum_{b=1}^M w_b \hat{S}_b(t), \quad w_b > 0, \sum_{b=1}^M w_b = 1 \tag{5}$$

Four merits of inferring a product’s run-off profile from several run-off estimates are: firstly, utilising multiple run-off estimates to infer the run-off of a class of business increases the credibility of results. This, however, comes at the cost of more computational resources. That is, it takes longer to process multiple run-off profile estimates. It also requires more data storage. Secondly, tracking run-off profiles on a class of business from several base dates helps to monitor changes in run-off experience. For indeterminate maturity deposits, a run-off profile estimated from a more recent base date takes account of: additional deposits; new customers; and changes in the composition of customers. Thirdly, a setting akin to simulation is achieved without the influence of additional deposits on calibrating the run-off profile model of bank liabilities. This is ideal since additional deposits are uninformative about the time that liability positions are retained on a bank’s balance sheet. In contrast, Kalkbrenner and Willing (2004), Neu (2007), Bardenhewer (2007), and Vento and La Ganga (2009) calibrate their models on the time series of an indeterminate maturity product’s

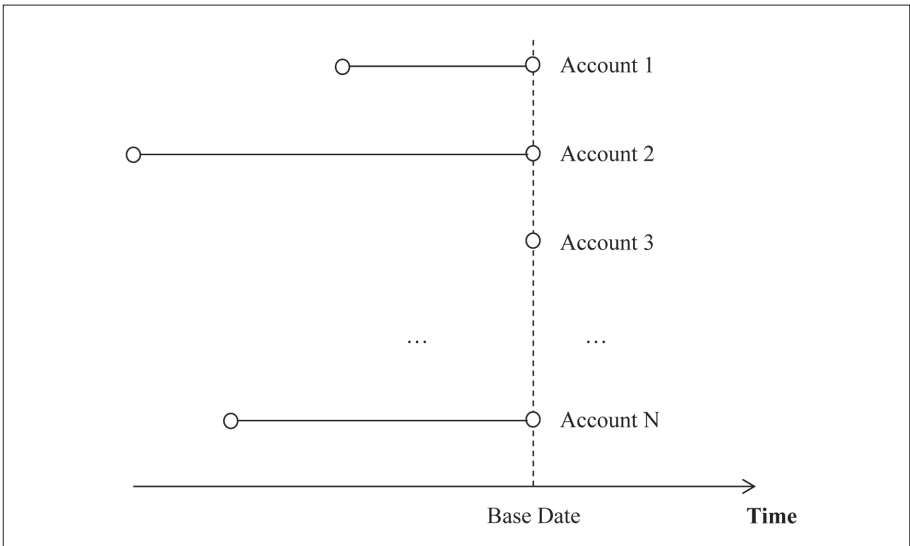


FIGURE 2 Time origin examples on base date

volume. Such an approach has the limitation that additional deposits influence the value of parameters such as volatility, which have a significant influence on run-off profiles resulting from their models. Fourthly, it enables calibrating state-dependent run-off profiles. This is so because run-off profile estimates can be grouped according to the liquidity scenario associated with the base date. This is discussed further in the subsequent section.

3.5 Incorporating Liquidity Scenarios when calibrating Funding Liquidity Risk Models

Up to this point, liquidity scenarios have been assumed constant. Funding liquidity risk assessment, however, is scenario specific (Basel Committee on Banking Supervision, 2010). Cash-flows stemming from existing assets and liabilities considerably depend on the underlying funding liquidity risk scenario since it is a major influence on the behaviour of the firm and its stakeholders (Neu, 2007; Matz, 2007). While the influence of scenarios on a firm's funding liquidity risk profile is widely acknowledged, the same explicit recognition appears lacking in the literature when calibrating models employed to project cash-flows.

It is difficult to envisage all funding liquidity risk scenarios that can possibly be considered when analysing funding liquidity risk. Moreover, the importance of analysing particular liquidity scenarios varies by firm and time. To thus specify scenarios applicable to a broader spectrum of firms, this study follows Matz (2007), the Basel Committee on Banking Supervision (2010), and the Committee of European Banking Supervisors (2009) who group liquidity scenarios as stemming from either bank-specific factors or market-specific factors. Examples of bank-specific factors include: credit rating downgrade; significant operational loss or credit risk event; and negative market rumours about the firm (Basel Committee on Banking Supervision, 2010). Market-specific factors include: disorder in capital markets, economic recession, and payment system disruption (Matz, 2007).

Fig. 3 shows the resulting four funding liquidity risk scenarios arising from the grouping of funding liquidity risk factors. Examples of terms to describe scenario 1, in Fig. 3, include: 'business as usual' and 'normal operating environment'. Scenario 2 can be described as an 'idiosyncratic-stress scenario'. Scenario 3 can be termed a 'market stress scenario' while scenario 4 describes a state where funding liquidity stress is arising from both market-specific and bank-specific factors. Bank regulators are particularly concerned with a bank's liquidity metrics under scenario 4.

The time spent in a particular scenario is random. Thus, it is impossible to *ex-ante* establish which scenarios will be observed over an experiment period. For example, it could be that over the experiment period, most of the time is only one scenario, with less credible time observed for other scenarios. In this case, only meaningful inference can be made to one scenario. Nevertheless, *ex-post*, one can make reasonable judgement of run-off profiles for other states by making inference relative to the state where credible time spent was experienced.

		Bank-specific Stress Scenario	
		No	Yes
Market stress scenario	No	Scenario 1	Scenario 2
	Yes	Scenario 3	Scenario 4

FIGURE 3 Funding Liquidity Scenario Matrix

4. NUMERICAL ILLUSTRATION AND CASE STUDY

The objective of this section is threefold:

- (1) To illustrate how account data is collected in a way to infer that account’s run-off;
- (2) To show how the run-off of a class of business is inferred from aggregating account-level data; and
- (3) To present a case study on how the run-off profile of a savings product was derived from a proprietary bank’s dataset.

The derived run-off profile, however, cannot be generalised to other banks. Run-off profiles on a product vary across banks partly because of differences in client profile. Therefore, the results are only to illustrate the method of estimating run-off profiles.

4.1. Numerical Illustration of processing Account-level Data

Table 1 shows an example of daily data collected on an account over 14 days. Here, the liquidity scenario is constant over the 14 days (see column entitled ‘Liquidity State’). A liquidity scenario relates to a specific day, not an account. Thus, on a specific day, the liquidity scenario is the same for all accounts. The column ‘Liquidity State’ shows liquidity scenarios as defined in Fig. 3. For example, State 1 corresponds to ‘no bank-specific stress’ and ‘no market stress’ scenario.

To illustrate the time origin concept, consider day 9 in Table 1. The local maximum account balance when moving back in time is 1020. The corresponding furthest recorded day associated with the local maximum account balance, i.e. the time origin, is day 2. This explains why, on base day 9, the run-off starts being tracked on day 2.

Table 2 shows the same example as in Table 1, but the liquidity scenario changes over the 14 days. Specifically, on days 1 to 8, the liquidity scenario is State 1. Thereafter, the liquidity scenario is State 2. This change in liquidity scenario results in truncated run-off data arrays. For example, consider base day 8. While the run-off data on base

TABLE 1 Example of an account's contribution to determining run-off profiles on various base days

Day	Data Collected		Base Day															
	Account Balance	Time Origin	Liquidity State	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
1	1000	1	1	1 000														
2	1020	2	1	1 000	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	
3	1020	2	1	1 000	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	
4	1020	2	1	1 000	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	
5	1020	2	1	1 000	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	
6	1020	2	1	1 000	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	
7	1020	2	1	1 000	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	
8	800	2	1	800	800	800	800	800	800	800	800	800	800	800	800	800	800	
9	800	2	1	800	800	800	800	800	800	800	800	800	800	800	800	800	800	
10	800	2	1	800	800	800	800	800	800	800	800	800	800	800	800	800	800	
11	500	2	1	500	500	500	500	500	500	500	500	500	500	500	500	500	500	
12	2000	12	1	500	500	500	500	500	500	500	500	500	500	500	500	2 000	2 000	2 000
13	2000	12	1	500	500	500	500	500	500	500	500	500	500	500	500	2 000	2 000	2 000
14	2000	12	1	500	500	500	500	500	500	500	500	500	500	500	500	2 000	2 000	2 000

NOTES:

- 1) The column 'Liquidity State' shows liquidity scenarios as defined in Fig. 3. For example, State 1 corresponds to 'no bank-specific stress' and 'no market stress' scenario.
- 2) On a specific base day, the run-off on an account is tracked from the corresponding time origin.
- 3) Eq. (2) is used to calculate the monetary values shown under run-off base day columns.

TABLE 2 Example of an account's contribution to determining state-dependent run-off profiles on various base days

Day	Data Collected		Base Day														
	Account Balance	Time Origin	Liquidity State	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	1000	1	1	1 000													
2	1020	2	1	1 000	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020
3	1020	2	1	1 000	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020
4	1020	2	1	1 000	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020
5	1020	2	1	1 000	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020
6	1020	2	1	1 000	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020
7	1020	2	1	1 000	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020	1 020
8	800	2	1	800	800	800	800	800	800	800	800	800	800	800	800	800	800
9	800	9	2														
10	800	9	2														
11	500	9	2														
12	2000	12	2														
13	2000	12	2														
14	2000	12	2														

NOTES:

- 1) The column 'Liquidity State' shows liquidity scenarios as defined in Fig. 3. For example, State 1 corresponds to no bank-specific stress scenario and no market stress scenario.
- 2) On a specific base day, the run-off on an account is tracked from the corresponding time origin.
- 3) Eq. (2) is used to calculate the monetary values shown under run-off base day columns.
- 4) Observations under base days 1 to 8 are input to determine the run-off profile associated with State 1. Note, transiting between liquidity states truncates observed run-off data. That is why observations under base days 1 to 8 end on day 8. Data under base days 9 to 14 (i.e., data on a grey background) are input to determining the run-off profile associated with State 2.

day 8 spans from day 2 to 14 in Table 1, it only spans from day 2 to day 8 in Table 2. The reason is that, in Table 2, data from day 9 onwards relates to State 2. A change in the liquidity scenario also alters the time origin. For example, on day 9, the time origin is 9 because the previous balance history relates to run-off data associated with State 1.

Table 3 presents tables that show how account-level data is aggregated to estimate a product's run-off profile. The illustration shows datasets collated in respect of M base days. On base day j , the count of customer accounts is denoted by $N(j)$. Note that $N(j)$, where $(1 \leq j \leq M)$, may differ across j . This results from the net effect of accounts being opened and existing accounts being closed.

In Table 3, information on Account number 1 is derived from Table 1. To illustrate, consider Table 1's data array on the column corresponding to base day 2. This data array is the same information shown in Table 3, under Account number 1, on the table showing bank positions on balance sheet for base day 2.

Table 3 data is in a form where the population count, censored count and withdrawal (deaths) count are known at exact times. Thus, standard survival techniques, such as the product limit estimator by Kaplan and Meier (1958), can be employed to estimate the probability that a position on a product is retained on a bank's book for a specific period. In a discrete time framework, as shown in Table 3, censoring times often coincide with the withdrawal times. At such times, this study adopts the convention of assuming that withdrawals precede censoring.

Fig. 4 summarises the use of data in Table 3. For each base day, a survival model is employed to produce a run-off estimate. There is also a corresponding liquidity scenario associated with each base day. Run-off estimates associated with a specific scenario are then utilised to estimate the run-off profile under that scenario.

While Table 1, Table 2 and Table 3 illustrate a case where each observation day is also a base day, this need not necessarily be the case in practice. There can be fewer base days than observation days. For example, base days may be set weekly while data is observed daily.

4.2. Case Study for Determining Run-off Profile of a Savings Product

Proprietary data from a Southern Africa bank was used to estimate the run-off profile of one of its savings account products. Balances on the savings accounts were tracked daily over a period of 460 working days, excluding public holidays and Sundays. Only working days were used because human resources in the Risk Management department were available to capture the liquidity state associated with a specific day. Liquidity scenarios were observed over the 460 working days and specified as shown in Fig. 3. During the observation period, the balance on each active account was observed at the end of each working day. At the same time, the time origin of that balance was recorded. Of the total observation days, 96.1% (442 successive working days) were from a 'business as usual' liquidity scenario, i.e. Scenario 1 in Fig 3.

Fig. 5 shows the run-off profiles on the savings product derived from 130 base

TABLE 3 Data Format to Apply Survival Analysis

Bank Positions on Balance Sheet (Lives on Risk of Death)										Withdrawal (Death)								Non-withdrawal Decrement (Censored)											
Base Day 1										Base Day 1								Base Day 1											
Acc. #	0	1	2	3	4	5	6	7	8	Acc. #	1	2	3	4	5	6	7	8	...	Acc. #	1	2	3	4	5	6	7	8	...
1	1000	1000	1000	1000	1000	1000	1000	1000	800	1	0	0	0	0	0	0	200	0	...	1	0	0	0	0	0	0	0	0	...
2	2	2
N(1)	N(1)	N(1)
Base Day 2										Base Day 2								Base Day 2											
Acc. #	0	1	2	3	4	5	6	7	8	Acc. #	1	2	3	4	5	6	7	8	...	Acc. #	1	2	3	4	5	6	7	8	...
1	1020	1020	1020	1020	1020	1020	800	800	800	1	0	0	0	0	0	220	0	0	...	1	0	0	0	0	0	0	0	0	...
2	2	2
N(2)	N(2)	N(2)
...
Base Day 14										Base Day 14								Base Day 14											
Acc. #	0	1	2	3	4	5	6	7	8	Acc. #	1	2	3	4	5	6	7	8	...	Acc. #	1	2	3	4	5	6	7	8	...
1	2000	2000	2000	2000	2000	2000	1	0	0	1	0	0	2000	
2	2	2
N(14)	N(14)	N(14)
...
Base Day M										Base Day M								Base Day M											
Acc. #	0	1	2	3	4	5	6	7	8	Acc. #	1	2	3	4	5	6	7	8	...	Acc. #	1	2	3	4	5	6	7	8	...
1	1	1
2	2	2
N(M)	N(M)	N(M)

NOTE: 'Acc. #' denotes account number, a unique identifier of an account. Positions on balance sheet are derived from Eq. (2). The total number of accounts on base day j is denoted by $N(j)$. Account number 1 reflects data in Table 1.

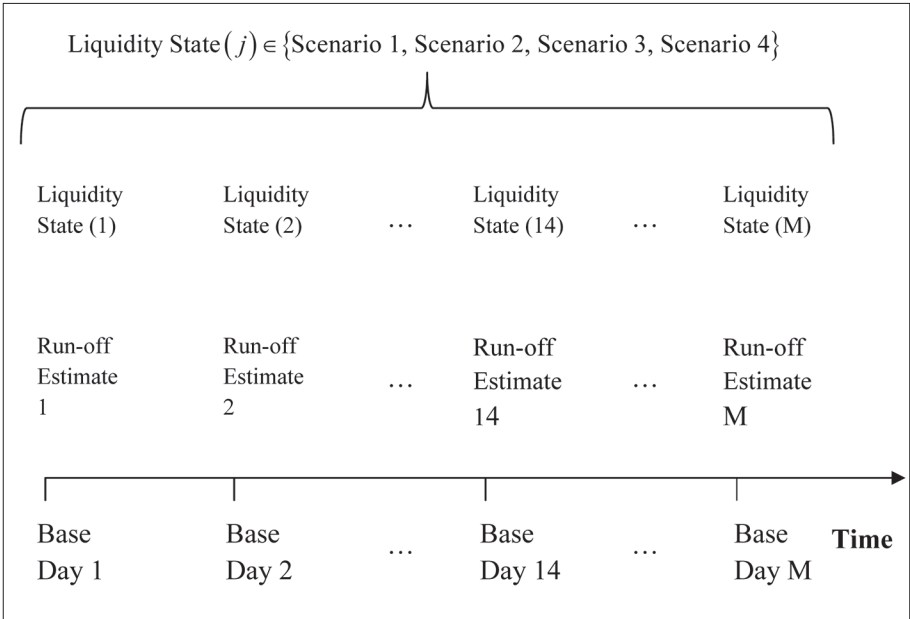


FIGURE 4 Run-off estimates and liquidity state illustration

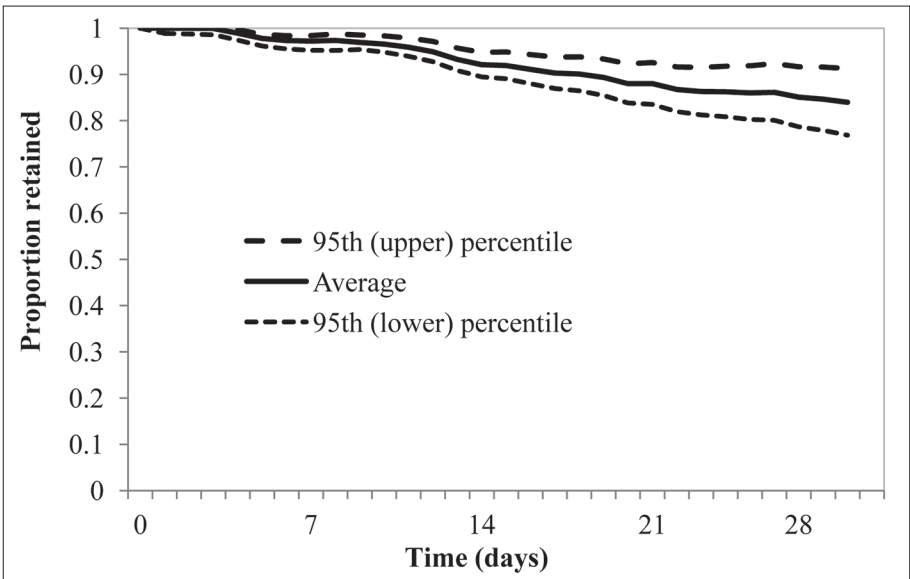


FIGURE 5 Run-off on savings product under a 'business as usual' liquidity scenario

days associated with the 'business as usual' liquidity scenario. These run-off profiles were estimated using the product limit estimator by Kaplan and Meier (1958). The dashed lines represent the 95th upper and lower percentile of the 130 run-off estimates generated during the 460 observation days. The continuous line, in Fig. 3, represents the weighted average run-off profile, calculated using Eq. (5). The average was based on a weighting process that assigned a higher weight to run-off estimates from more recent base days. The reasoning behind doing so was that, for predictive purposes, relatively recent data is arguably more informative than data further in the past.

5. CONCLUSION

This paper developed a non-parametric survival model of time that positions on indeterminate maturity products are retained on a bank's book. An algorithm to determine the time origin of bank positions provided the means of applying lifetime modelling techniques. By equating the survival function to the run-off profile, the model handles the problem of cash-flow timing uncertainty when measuring bank funding liquidity risk. Since liquidity risk is scenario specific, another focus of this study was determining state-dependent run-off profiles. This was achieved by taking cognisance of liquidity scenarios that prevailed in the past when calibrating run-off profiles.

The non-parametric and state-dependent run-off profile model has potential implications on how to evaluate the liquidity risk metrics of banks. For example, Basel III prescribes run-off rates on various classes of business when determining the liquidity coverage ratio, assuming a stress scenario. The debate on whether the prescribed run-off rates are reasonable has virtually been empirical without duly considering the method employed to derive state-dependent run-off rates. The methodology developed in this study shows that regulators can consider an internal model approach to determine a bank's liquidity risk metric.

This study can be extended in various ways. A useful area of research is studying correlations between run-off profiles of different assets and liabilities. Such correlations can be accounted for through copula functions. Another useful area of research is to model the evolution of an indeterminate maturity product's balance through separately projecting the run-off of existing business and that of future business. Finally, it is worthwhile to model changes in run-off profiles over time. This is analogous to developing a term structure of interest rate risk model to capture the dynamics of changes in the yield curve.

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