Pricing A Motor Extended Warranty With Limited Usage Cover

By

Fidelis T Musakwa

Agenda

1. Introduction to motor warranties.
2. Research Problem.
3. The Model.
4. Case Study.
5. Concluding remarks.

Introduction to Warranties

- Cover: Specified vehicle parts.
- Peril: Mechanical breakdown.
- Compensation: Parts and labour cost.
- Other Secondary Benefits: e.g., roadside assistance, hotel accommodation and car rental. `
- Service items: These are typically excluded from cover: e.g., oil filter, battery, brake pads and fuel filter.

Types of vehicle warranties

Vehicle Warranty

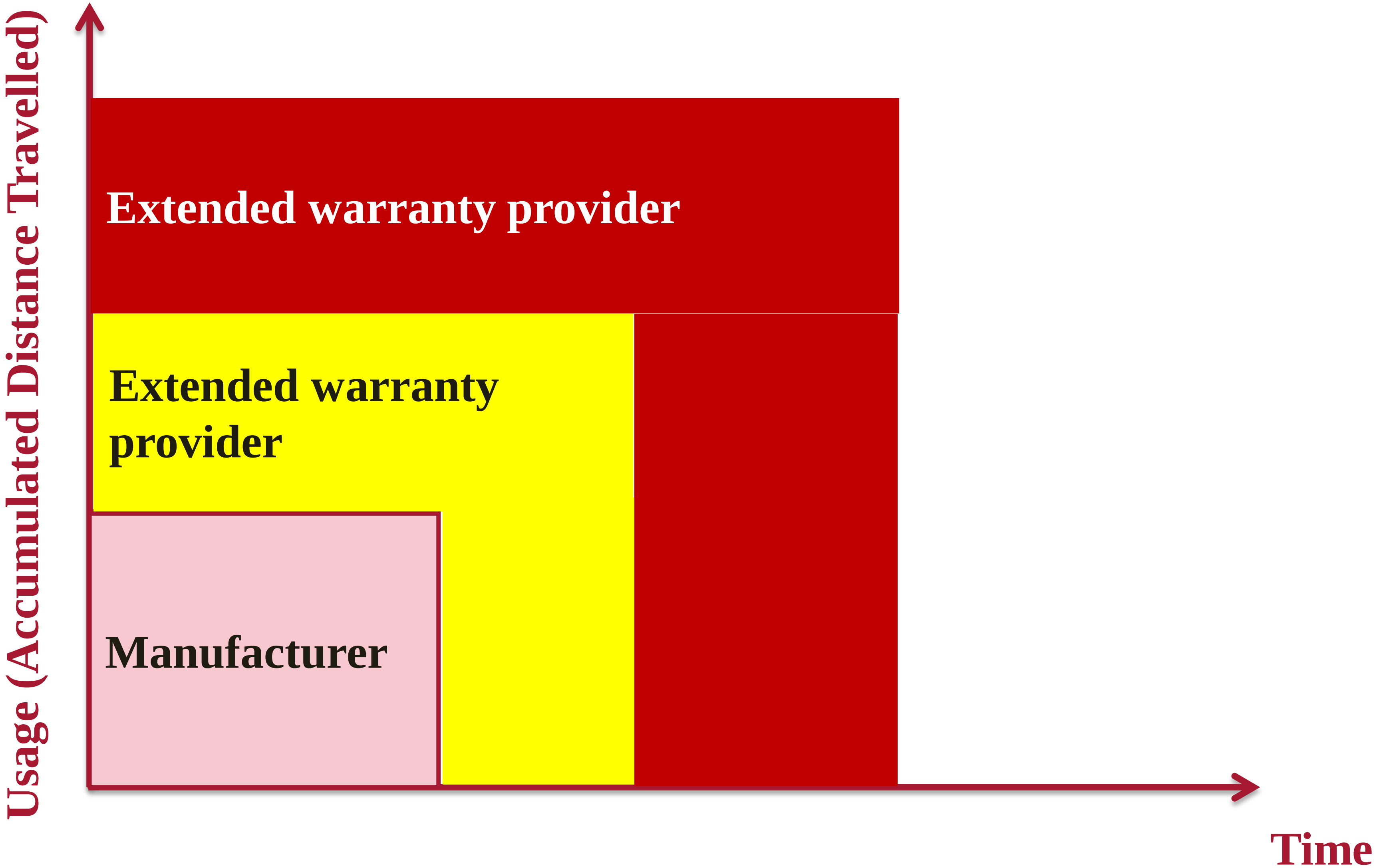
Base Warranty

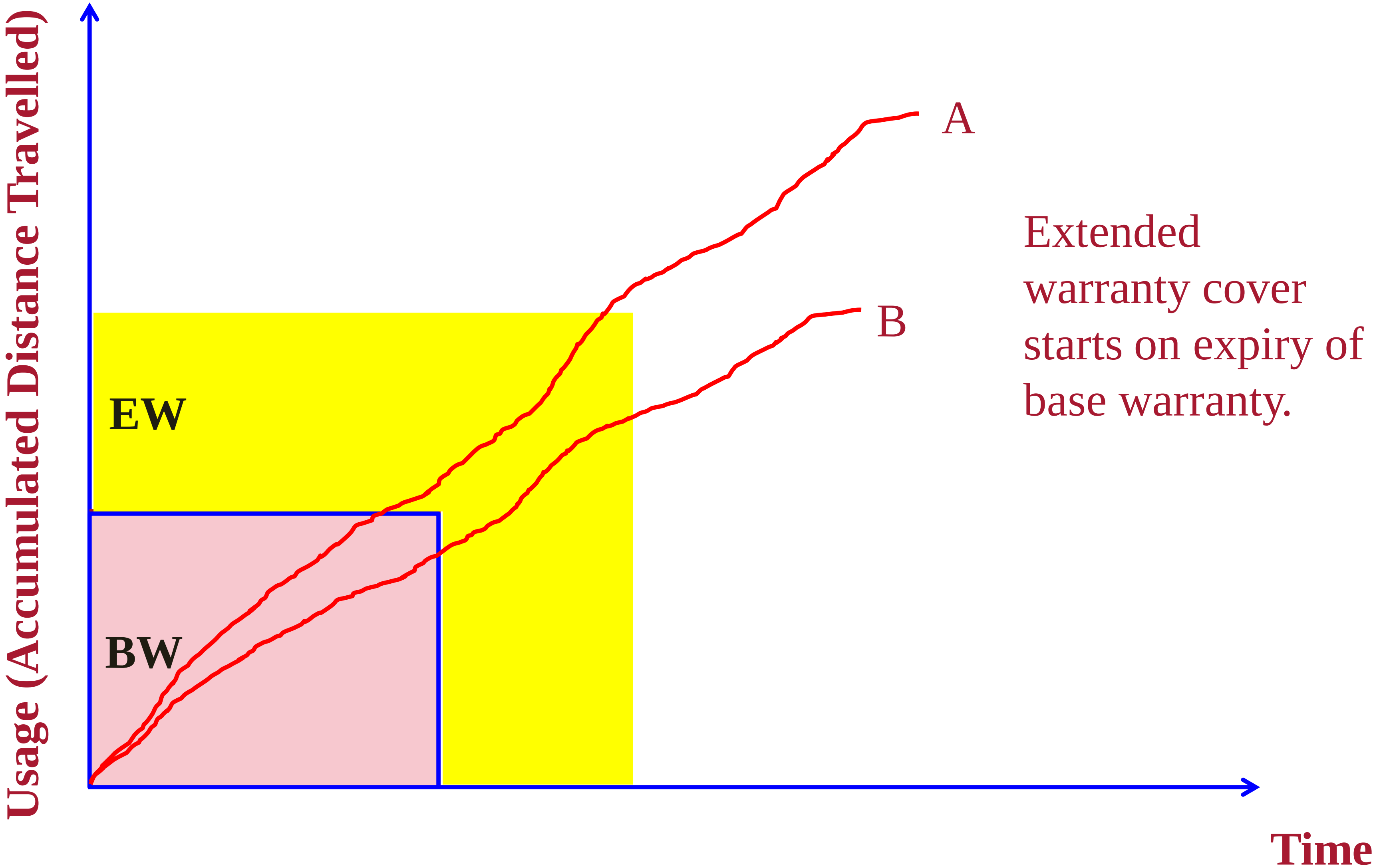
- Issued by manufacturer
- Tied to sale of new product
- Deemed as Non-insurance in most territories

Extended Warranty

- Issued by manufacturer and other third parties
- Customer has option to buy extended warranty
- Deemed as Insurance in most territories

Warranty cover





Extended warranty cover starts on expiry of base warranty.

Vehicle Exposure at Risk

- Exposure is unitised either as time or usage.
- Usage limit allowed for by projecting the vehicle population at risk via a usage rate distribution (Cheng and Bruce (1993); Rai and Singh (2005); Majeske (2007); Alam and Suzuki (2009); Su and Shen (2012); Wu (2012); Shahanaghi et al. (2013)).
- These studies rely on the **premise** that usage rate is **constant** on a vehicle but varies across vehicles.

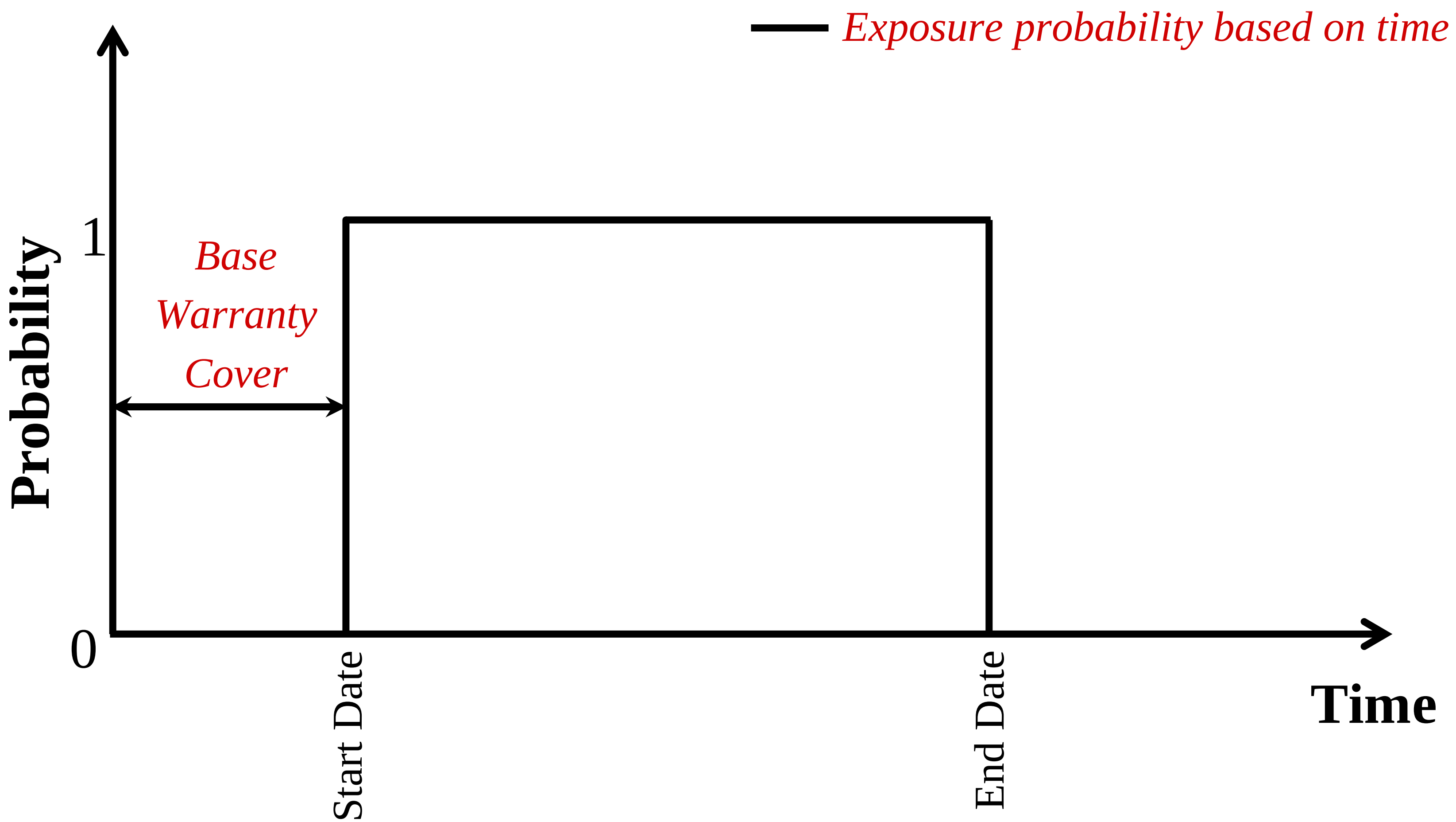
Research Questions

- If cover period is set on time and usage, how can a warranty provider estimate the probability of being on risk at a specific time in service?
- Can a usage rate distribution be reliably used to forecast the number of vehicles on risk?
- **Above all**, how do answers to the foregoing questions influence the ‘fair’ price of a motor extended warranty?

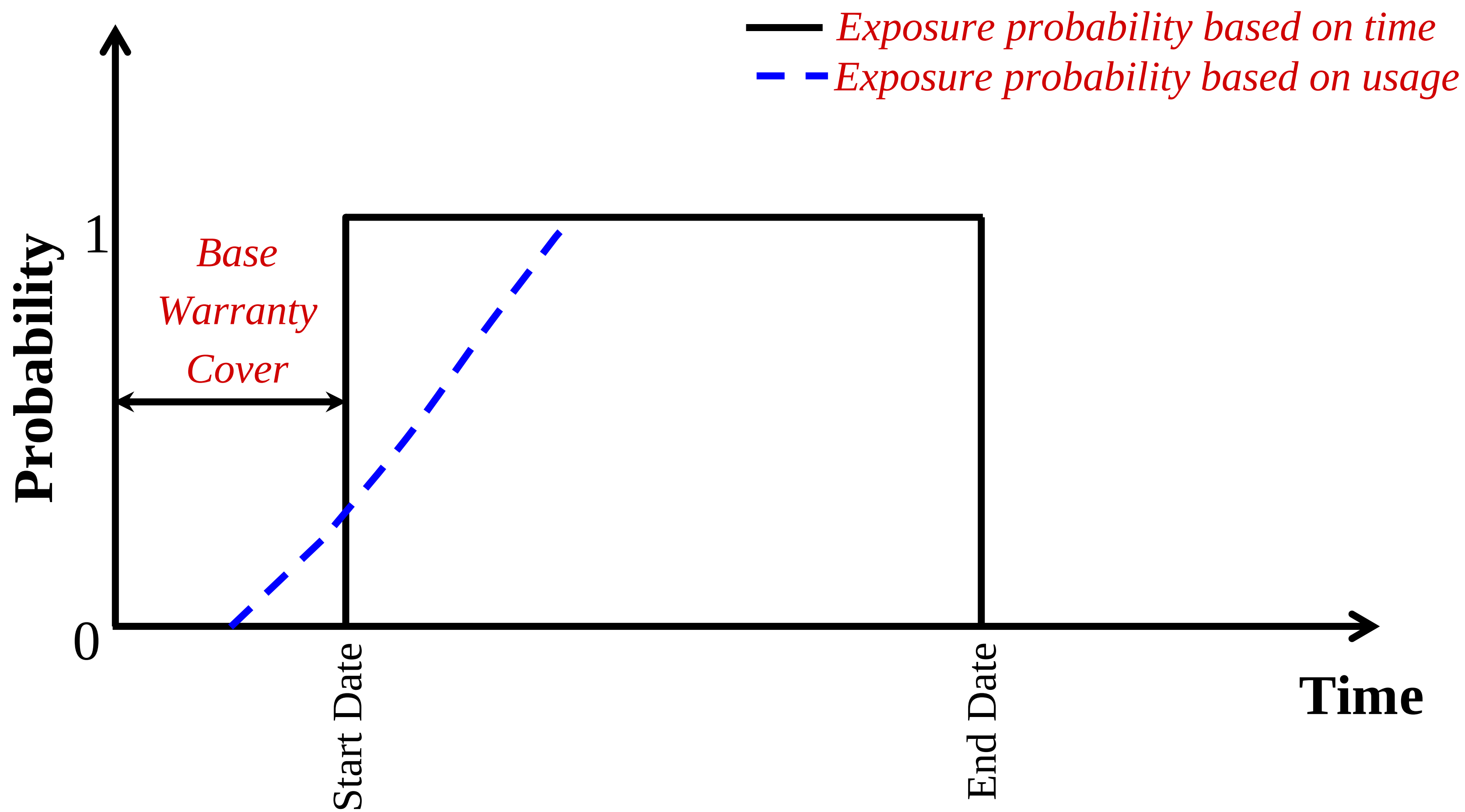
Research Contribution

- Develops an estimator of the probability that a warranty provider is on risk at a specific time in service.
- Employs a non-parametric interval-censored survival model to directly measure the probability distribution of time to accumulate a specific usage.
- Case study results suggest that employing a usage rate distribution to forecast the number of vehicles on risk can be misleading, especially on an extended warranty with a relatively high usage limit. This is despite observing that some positively skewed statistical distributions fit well to usage rate data.

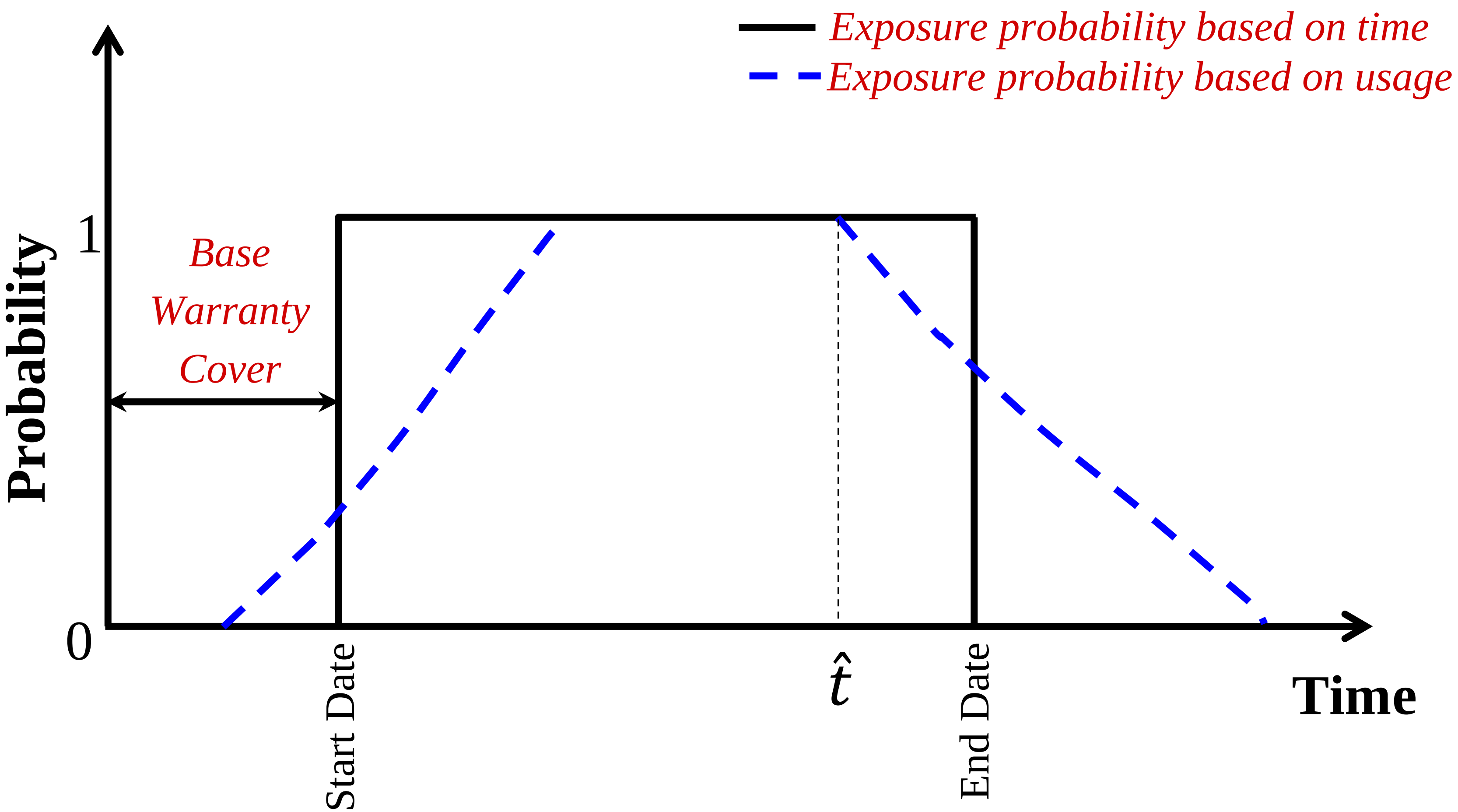
EW Provider Exposure Probability



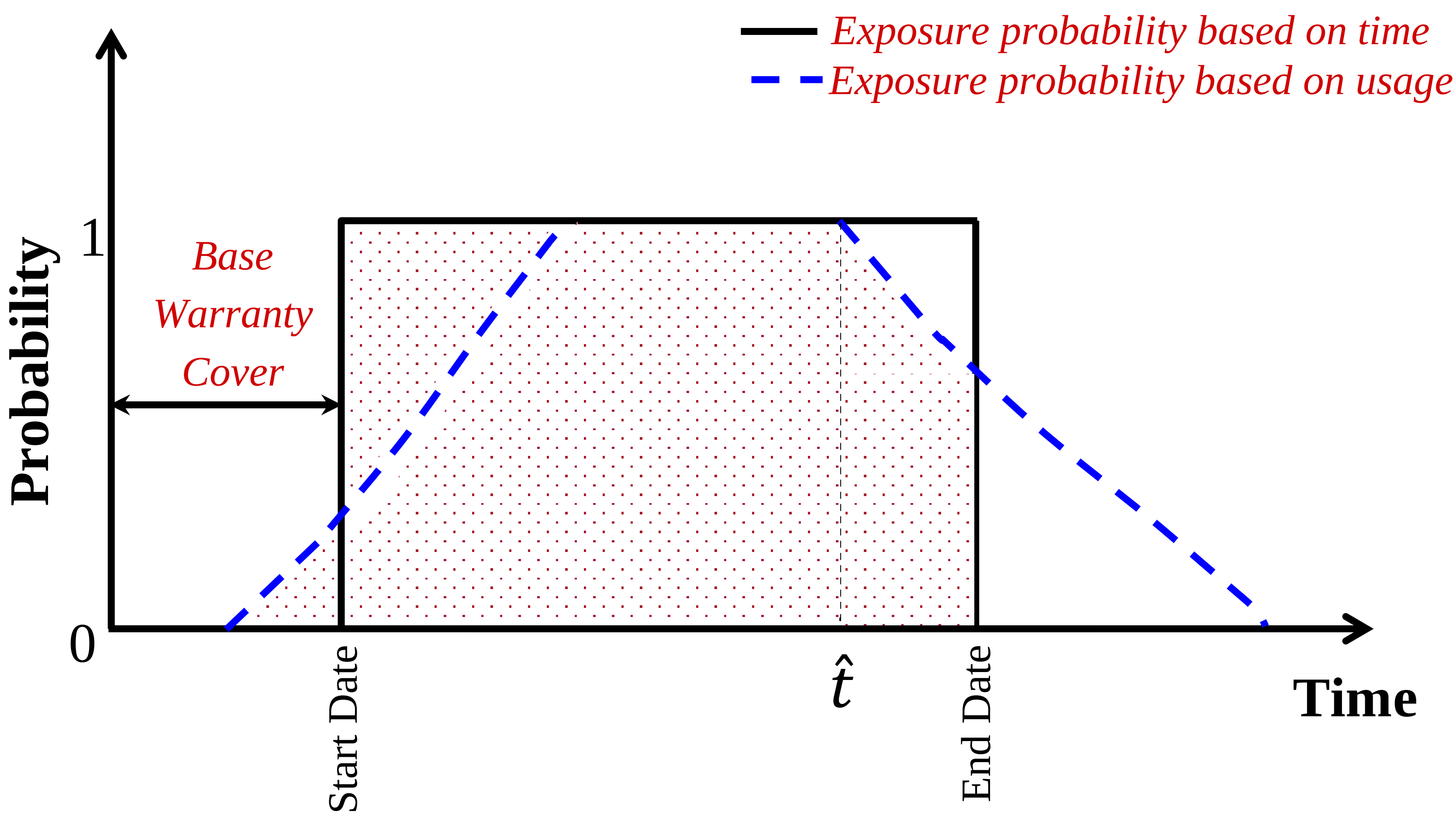
EW Provider Exposure Probability



EW Provider Exposure Probability



EW Provider Exposure Probability



Probability of being exposed to risk

$$p(t)=\begin{cases} p(t)^{Usage}, & t \le t^{BW} \\ 1, & (t^{BW} < t \le t^{EW}) \text{ and } (t < \hat{t}) \\ \min\left(p(t)^{Usage}, p(t)^{Time}\right), & (t^{BW} < t \le t^{EW}) \text{ and } (t \ge \hat{t}) \\ 0, & \text{Otherwise} \end{cases}$$

Where

t^{BW}=base warranty expiry date;
t^{EW} = Extended warranty expiry date

- Nonparametric interval-censored survival model.

$$S(t) = \Pr(T_U > t) = \Pr(U_t < U)$$

- Observed: $(L_i, R_i]$

$$\{T_U : L_i < T_U \leq R_i\}$$

Examples of Time Intervals

Interval for time to 400,000 kms: (L, R] = (2 months, 40 months].

Interval for time to 600,000 kms: (L, R] = (40 months, 67 months].

Interval for time to 800,000 kms: (L, R] = (67 months, ∞].

Nonparametric Maximum Likelihood Estimator

$$\text{Likelihood} = \prod_{i=1}^{n} \left[S(L_i) - S(R_i) \right]$$

- Closed form solution non-existent. So, iterative methods are used to estimate the survival function.
- For example:
  - Self consistent algorithm (Turnbull, 1976)
  - Expectation maximisation (EM) algorithm (Dempster et al., 1976)
  - Iterative convex minorant (ICM) (Groeneboom and Wellner, 1992)
  - EM-ICM algorithm (Wellner and Zhan, 1997).

Innermost intervals

Obs	L	R
1	L(1)	R(1)
2	L(2)	R(2)
3	L(3)	R(3)
.	.	.
.	.	.
.	.	.
N	L(N)	R(N)

$$\{L_i : i = 1, \ldots, n\} \cup \{R_i : i = 1, \ldots, n\}$$

$$0 < p_1 < q_1 < p_2 < q_2 < \ldots < q_m < \infty$$

$$\{(p_1, q_1], (p_2, q_2], \ldots, (p_m, q_m]\}$$

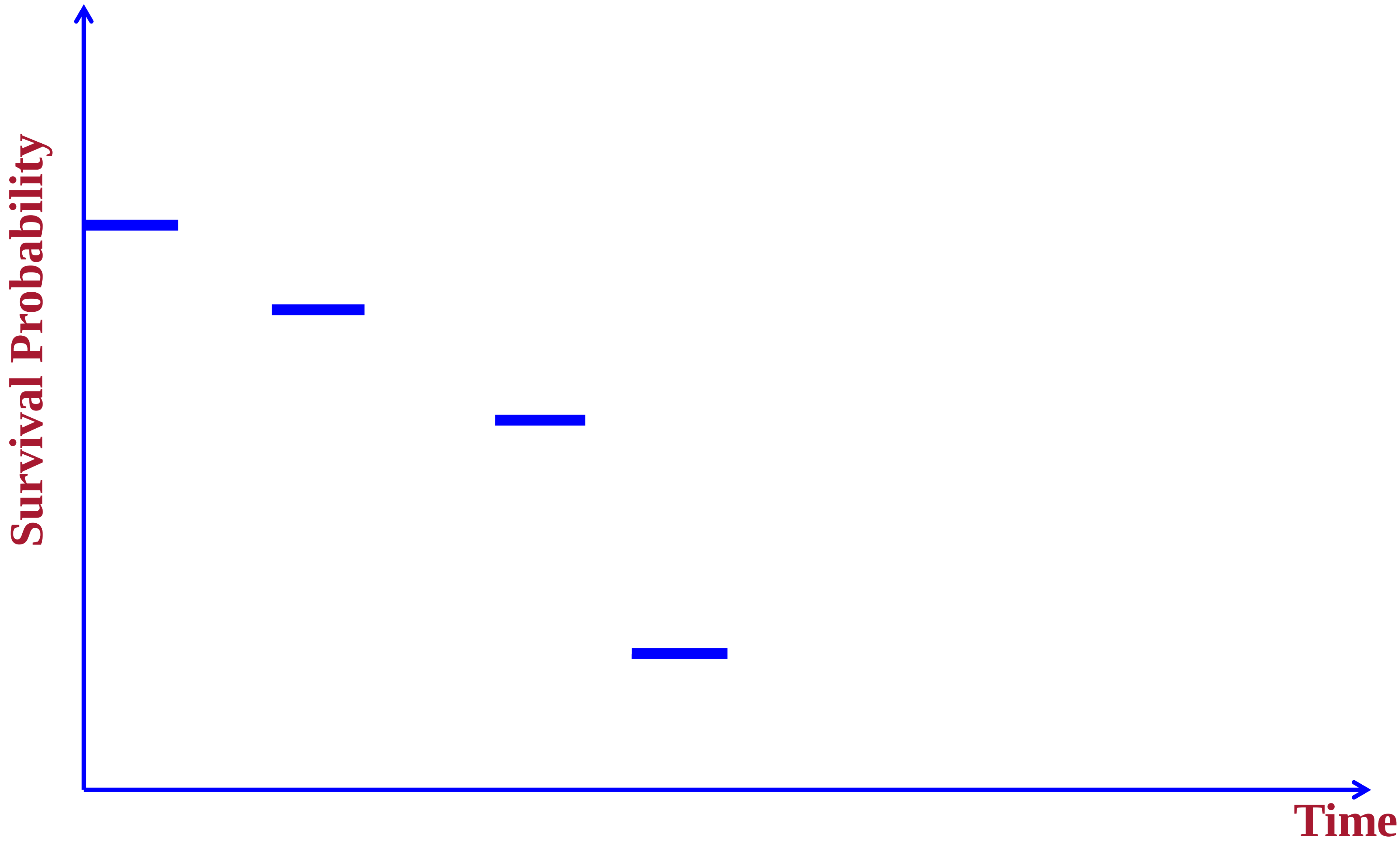
Innermost intervals: Numerical Example

Obs	L	R
1	20	32
2	14	30
3	45	∞
4	0	35

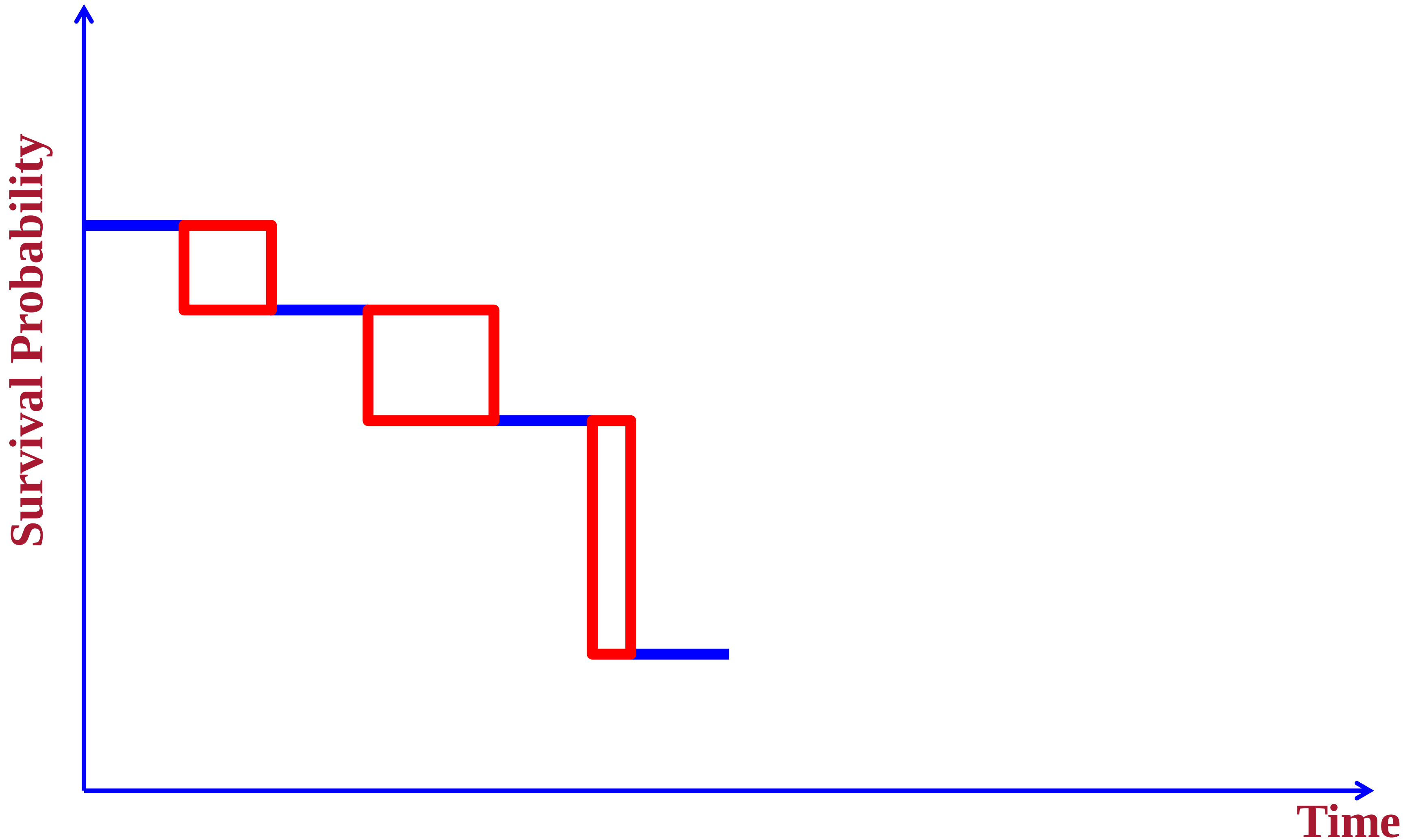
$$\{0, 14, 18, 20, 21, 30, 32, 35, 45, \infty\}$$

$$\{(14,18], (20,21], (30,32], (35,45]\}$$

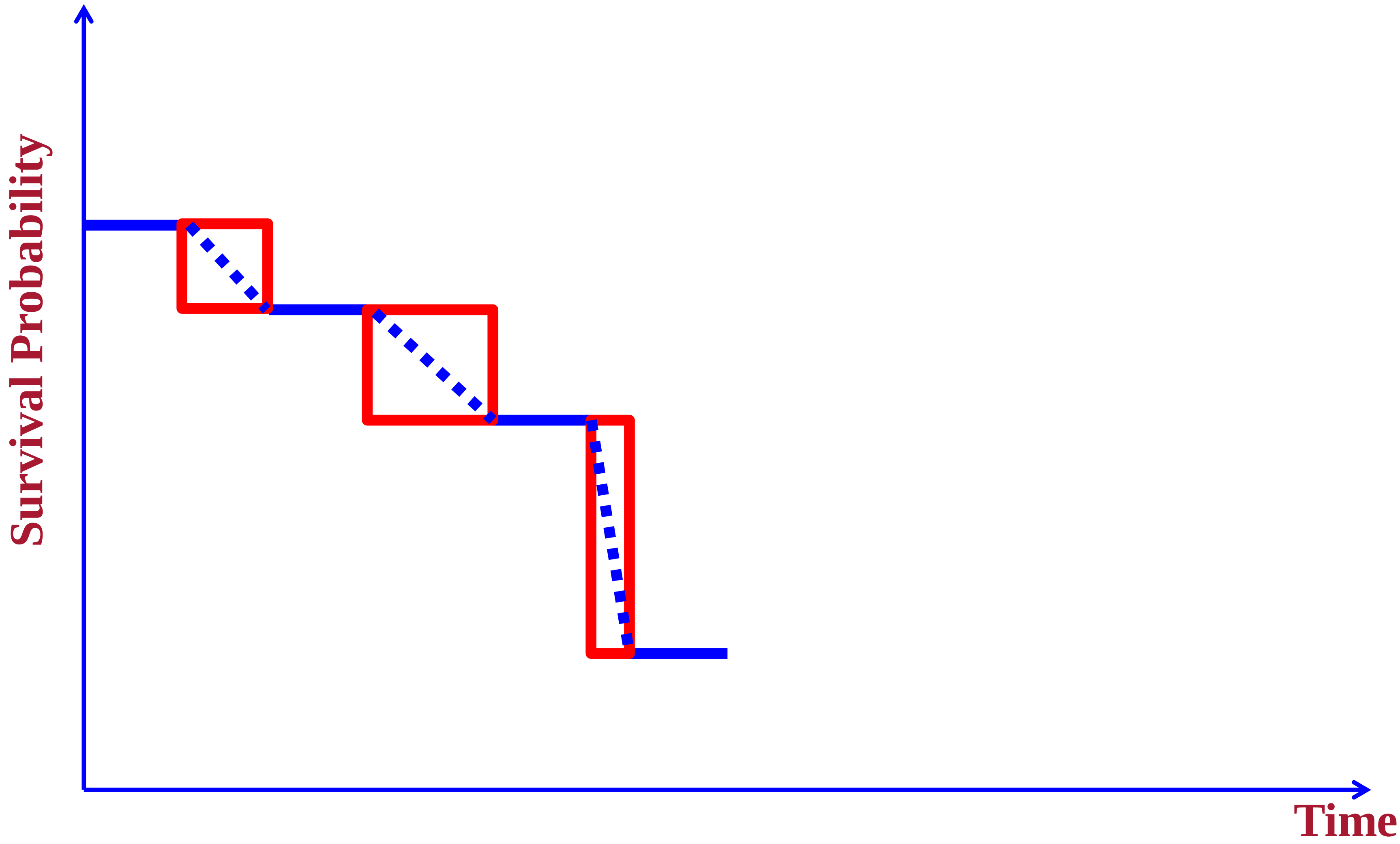
Interval-censored Survival Function



Interval-censored Survival Function



Interval-censored Survival Function



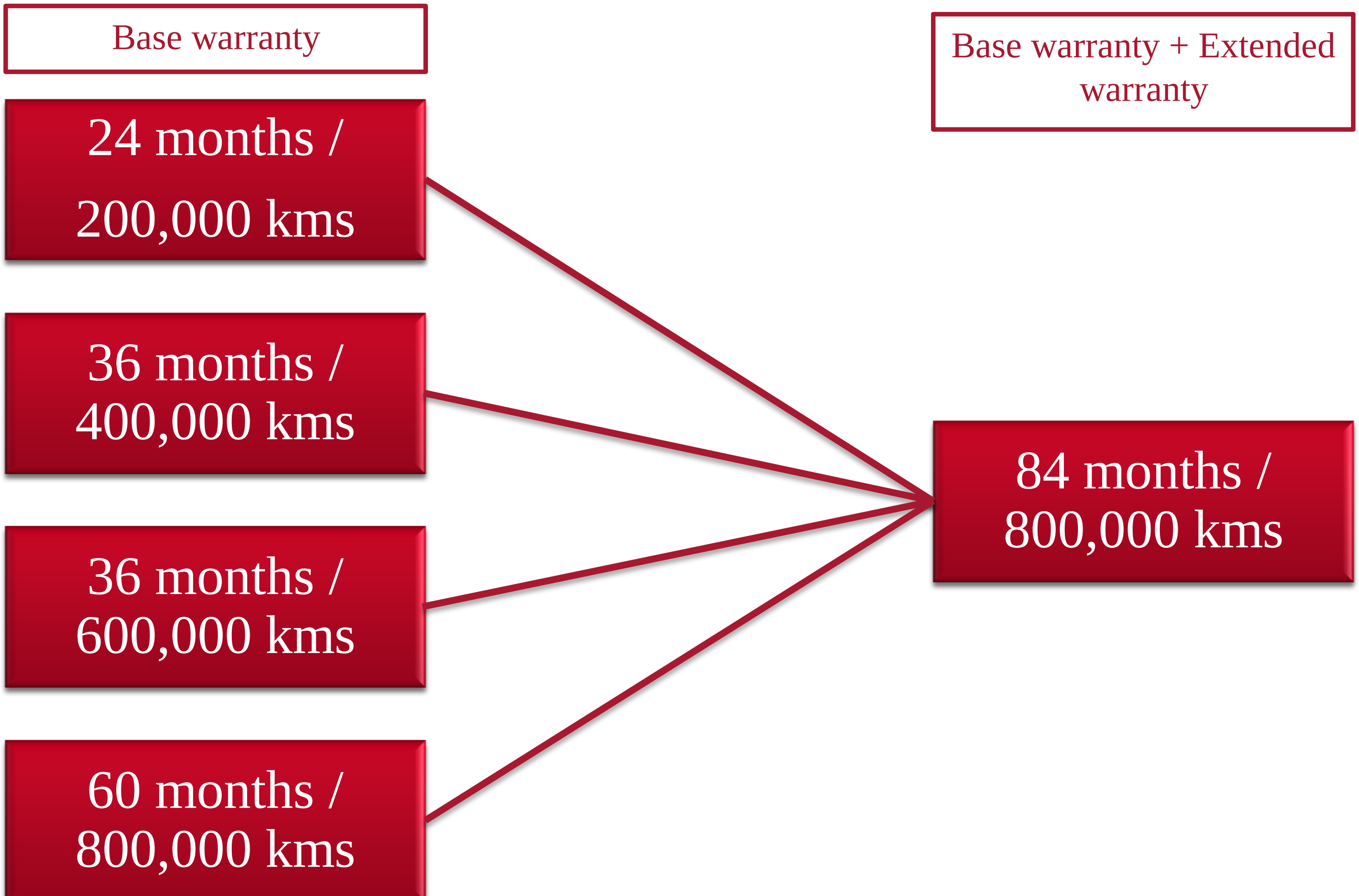
Warranty Risk Premium

$$\text{Risk premium} = \sum_t \left\{ \text{Cost per Exposure}(t) \times \text{Probability of Exposure}(t) \right\}$$

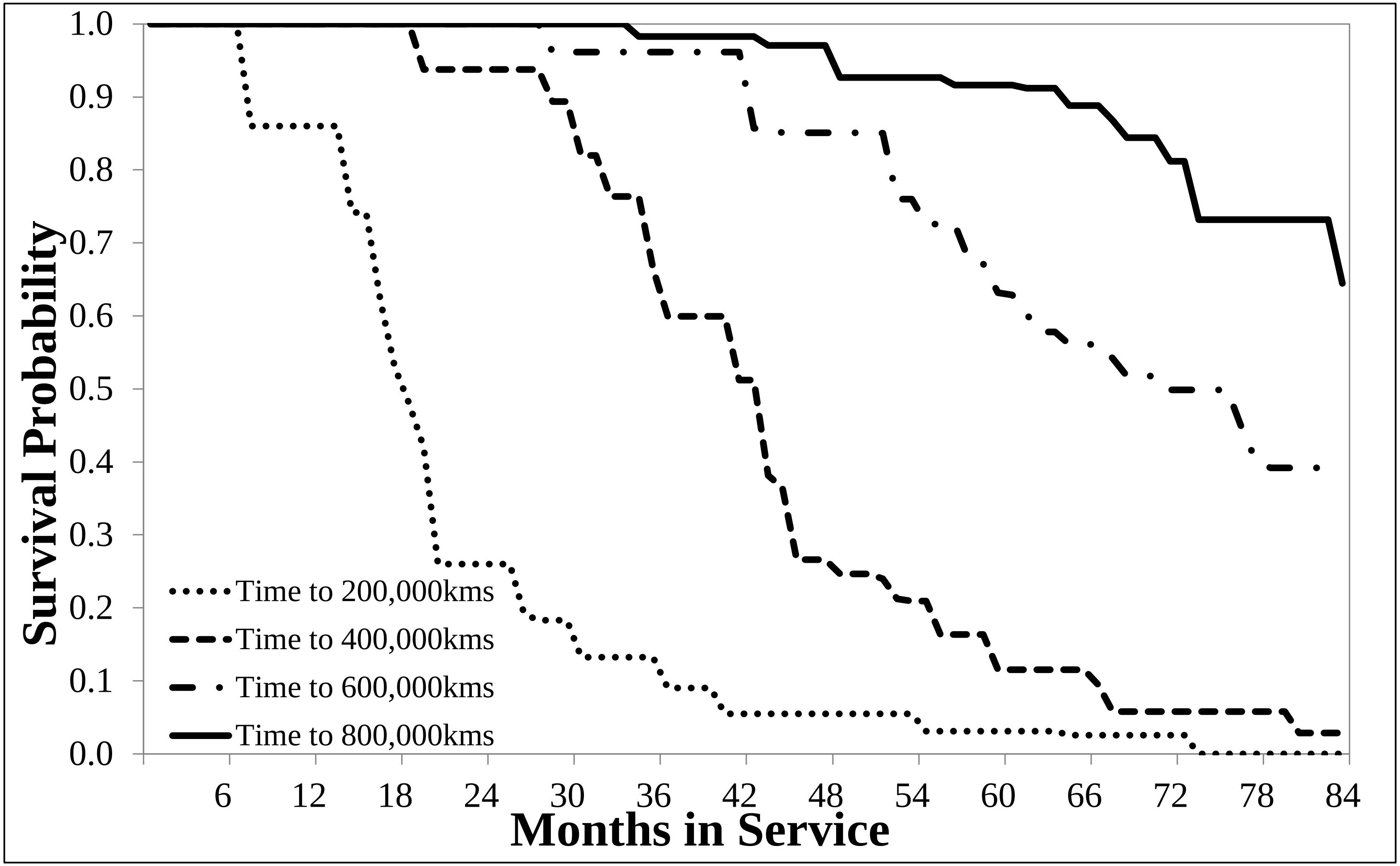
- Simulation can be employed to account for varied age and usage at extended warranty start date.

Data

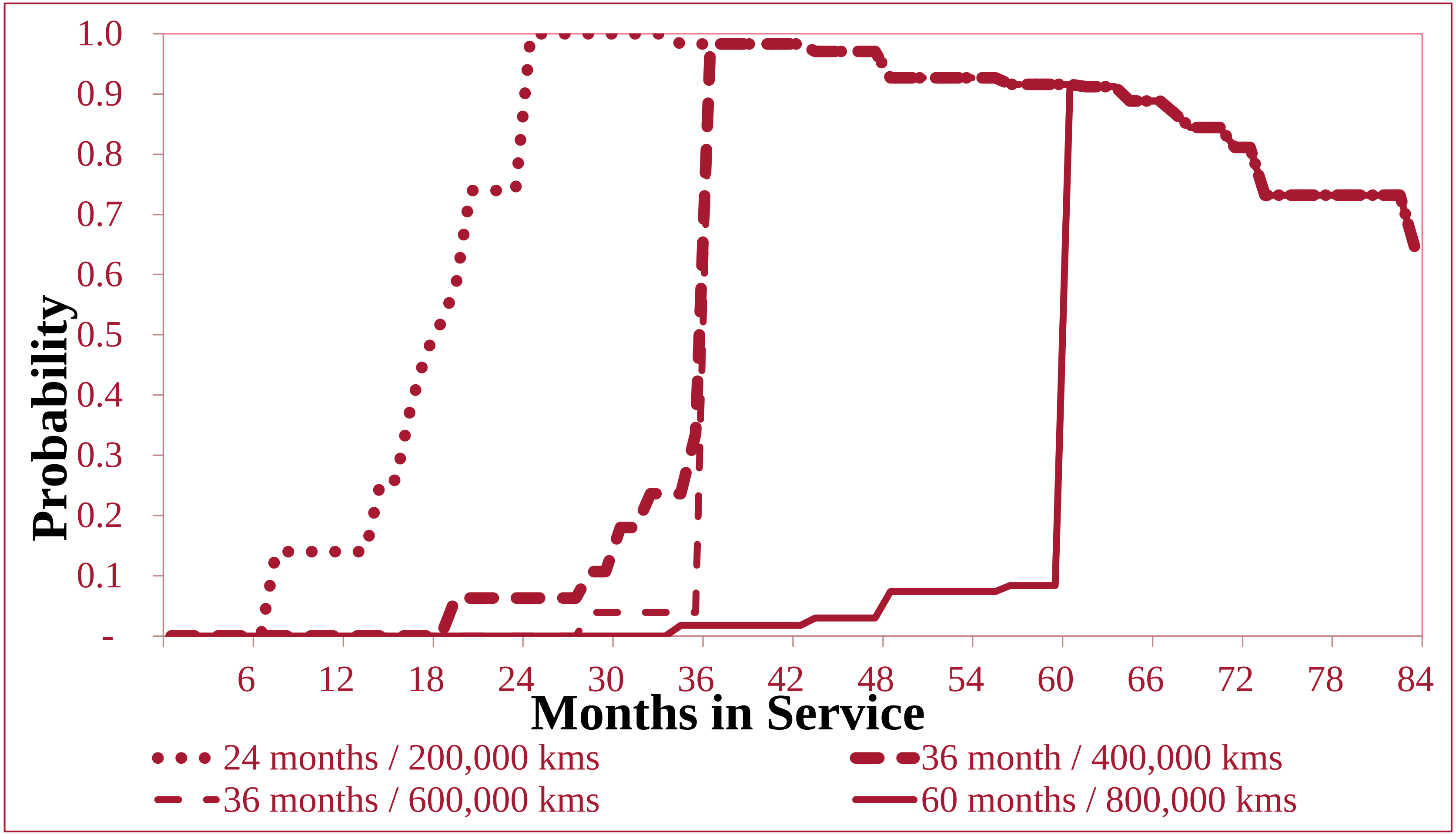
- Proprietary data of insurer located in South Africa
- Eligibility: heavy commercial truck involved in medium or long-haul operations.
- Usage data source: Withdrawals, claims, policy inception and maintenance records. Total: 857 trucks.
- Claims data source: only warranty data used to estimate claim severity and frequency.


Base warranty
Base warranty + Extended warranty
24 months / 200,000 kms
36 months / 400,000 kms
36 months / 600,000 kms
60 months / 800,000 kms
84 months / 800,000 kms

Time to Reach a Specific Usage



Pr. of EW Provider Being on Risk



Weighted Log-rank Test Results

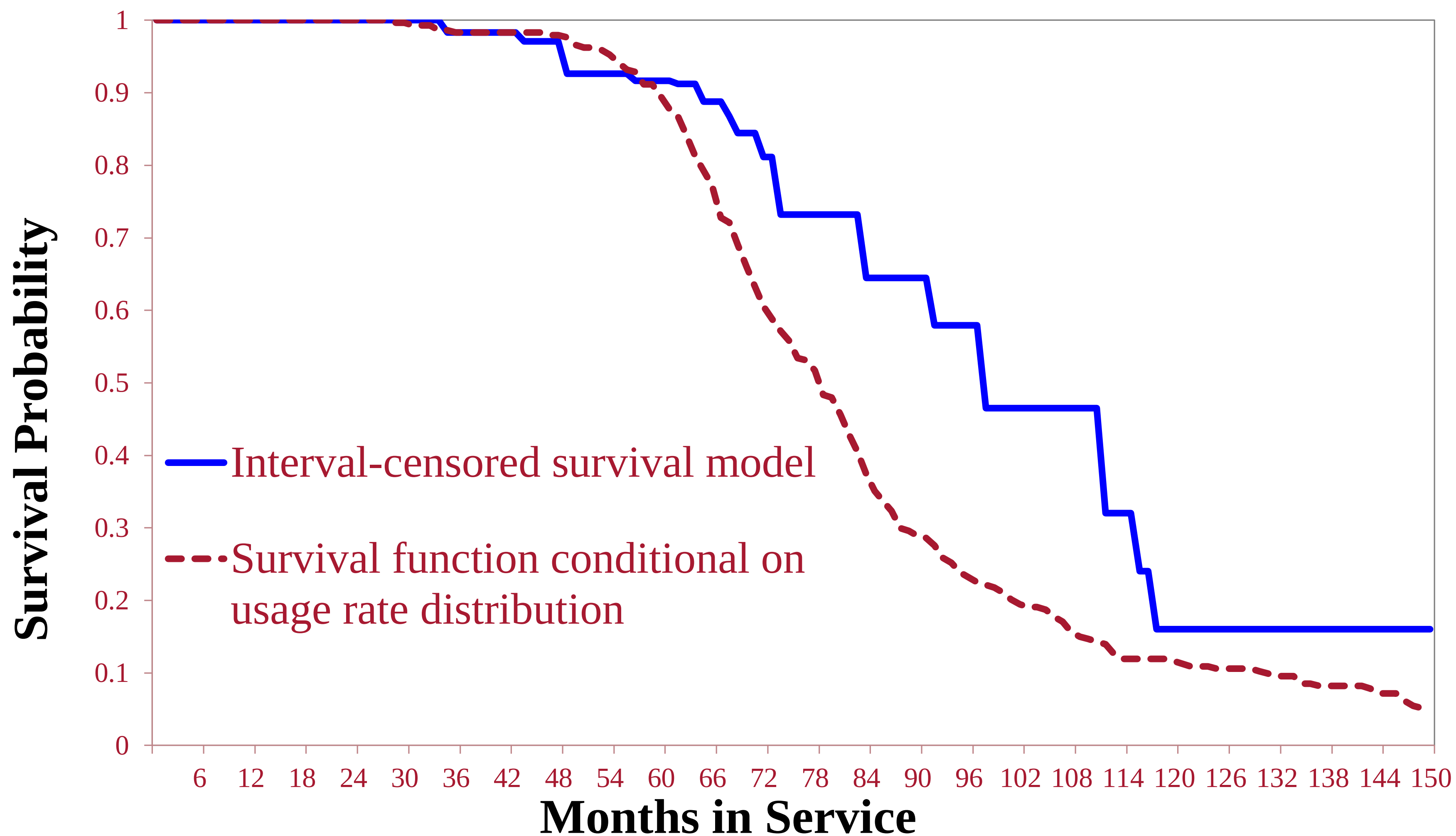
$$H_0 : S_{NPMLE}(t) = S(t|g(u)) \quad \forall t$$

$$H_{alt} : S_{NPMLE}(t) \neq S(t|g(u)) \quad \forall t$$

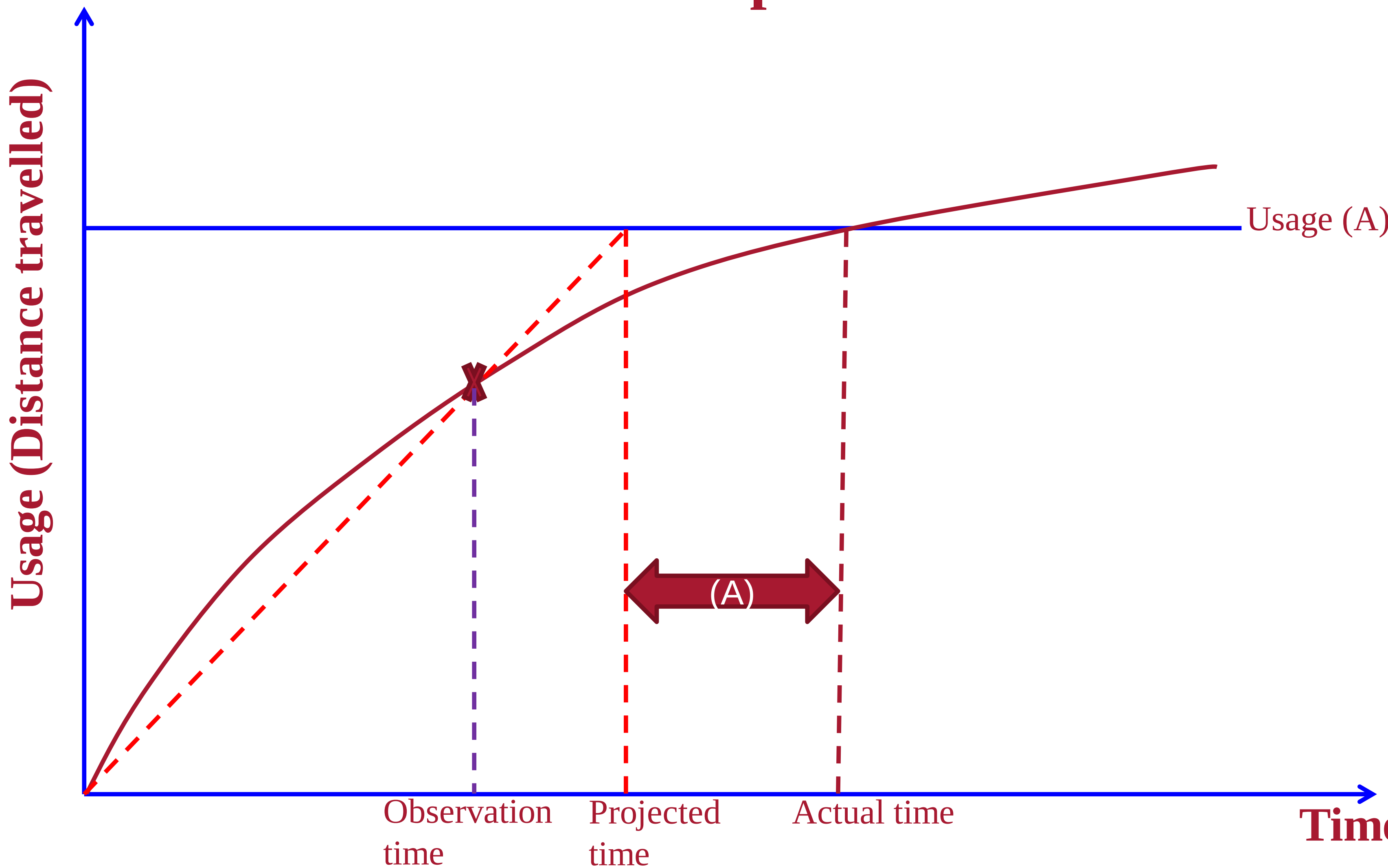
	Sun (1996) Score Test		Finkelstein (1986) Score Test		Fay and Shaw (2010) Monte Carlo Wilcoxon Test		
	Score Statistic	p-value	Score Statistic	p-value	Score Statistic	p-value	p-value 99% CI
Age at 200,000 kms	7.795	0.455	8.468	0.423	10.171	0.094	[0.093, 0.096]
Age at 400,000 kms	11.125	0.293	11.259	0.291	11.625	0.062	[0.061, 0.063]
Age at 600,000 kms	13.362	0.197	13.528	0.195	11.990	0.041	[0.040, 0.041]
Age at 800,000 kms	21.403	0.021	21.415	0.022	15.818	0.008	[0.007, 0.008]

ACTUARIAL SOCIETY OF SOUTH AFRICA

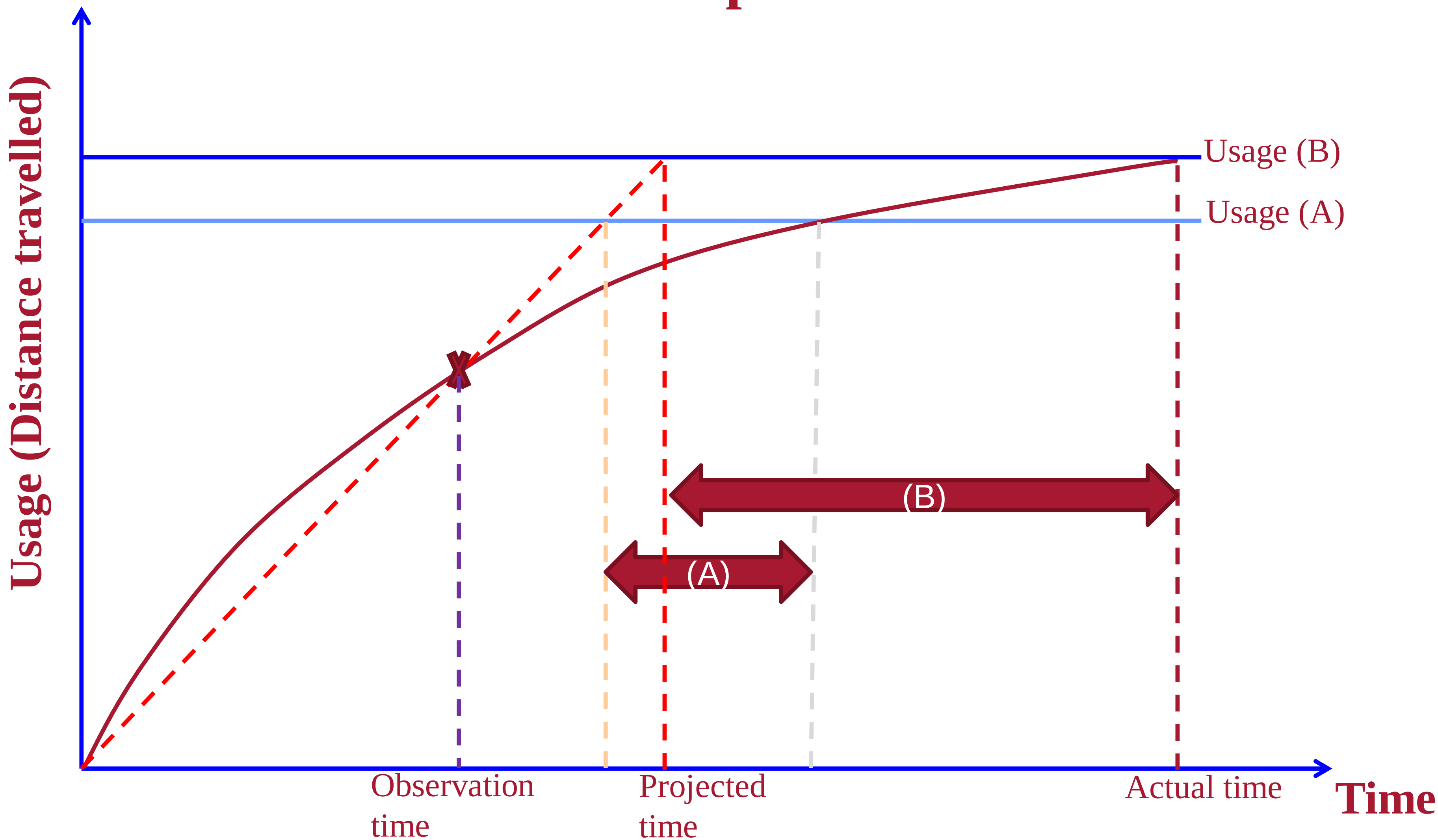
NPMLE vs Usage Rate Distribution (Time to 800,000 kms)



Plausible Explanation



Plausible Explanation



Goodness of Fit Test Results

Distribution	Parameter Estimates	Anderson-Darling Test Statistic	p-value
$Lognormal(\mu,\sigma)$	$\mu = 8.96;\ \sigma = 0.44$	0.38	0.39
$Gamma(\alpha,\beta)$	$\alpha = 8.24;\ \beta = 0.98 \times 10^{-3}$	0.27	0.66
$Weibull(\alpha,\beta)$	$\alpha = 3.22;\ \beta = 9{,}614.29$	0.25	0.73

- Results reaffirm that positively skewed statistical distributions fit well to usage rate data (Shahanaghi et al., 2013; Su and Shen, 2012; Jung and Bai, 2007; Kerper and Bowron, 2007; Majeske, 2007; Rai and Singh, 2005).

Summary

ACTUARIAL SOCIETY OF SOUTH AFRICA

- The effect of limiting usage on an extended warranty's risk premium can be captured through determining a provider's probability of being on risk at a specific time in service.
- Interval-censored survival models can suitably be employed to estimate exposure probabilities, especially given that extended warranty providers often have incomplete data on how usage accumulates with time.
- Employing a usage rate distribution can result in underestimating the risk premium.

Thank You

Any Questions?