HOW A SINGLE-FACTOR CAPM WORKS IN A MULTI-CURRENCY WORLD: RESULTS FROM THE LATEST RESEARCH

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HOW A SINGLE-FACTOR CAPM
WORKS IN A MULTI-CURRENCY
WORLD

ASSA Convention 2012

AFIR Colloquium 2012

David Wilkie:

“Why the capital asset pricing model fails in a multi-currency world”
Why use the CAPM?

- It’s an equilibrium model
- It assumes homogeneous expectations
- It’s a simple model
- Nobody has ever proved that it’s wrong
Global CAPM (GCAPM)

\[ E\{R_i\} = R_F + \beta^W_i \left[ E\{R_W\} - R_F \right] \]
International CAPM (ICAPM)

\[ E \{ R_i \} = R_F + \beta_i^W \left[ E \{ R_W \} - R_F \right] \]
\[ + \gamma_i^1 \left[ E \{ R_M^1 \} - R_F^1 \right] + \ldots + \gamma_i^C \left[ E \{ R_M^C \} - R_F^C \right] \]
What’s the problem?

• One currency’s point of view

• Different implied prices

• Different risk-free rates

• Variances from some sources more important than from others
Convergence problems

no. of constraints vs no. of unknowns
SFM-CAPM Assumptions

(1) Investors who measure their investment returns in currency c (i.e. ‘currency-c investors’) have indifference curves in mean–variance space, the means and variances being those measured in that currency.

(2) All investors, regardless of the currency in which they measure returns, have homogeneous expectations of the means, variances and covariances of:

(a) the returns in each currency on assets issued in that currency; and

(b) rates of strengthening of each currency.
The SFM-CAPM constraint

If the SFM-CAPM applies in a multi-currency world then, for any currencies c and e:

\[ K_{di}^c = K_{di}^e \]

where:

\[ K_{di}^c = \frac{\sigma_{\text{di},M}^c - \sigma_{d1,M}^c}{\sigma_{M,M}^c} \left( \mu_{M}^c - r_c \right) \]
SFM-CAPM mark 1

Minimise:

\[ D_\mu^2 = \frac{1}{Q_\mu} \left[ \sum_{c=1}^{C} \left\{ \sum_{i=2}^{n_c} \left( \hat{\mu}_{ci}^{(S)} - \hat{\mu}_{ci}^{(G)} \right)^2 \right\} + \sum_{c=2}^{C} \left( \hat{\mu}_{c}^{(S)} - \hat{\mu}_{c}^{(G)} \right)^2 \right] \]

subject to the constraints:

\[ \kappa_{di}^c = \frac{\hat{\sigma}_{di,M}^c - \hat{\sigma}_{d1,M}^c}{\hat{\sigma}_{M,M}^c} \left( \hat{\mu}_{M}^{c(S)} - r_c \right) = \frac{\hat{\sigma}_{di,M}^{e} - \hat{\sigma}_{d1,M}^{e}}{\hat{\sigma}_{M,M}^{e}} \left( \hat{\mu}_{M}^{e(S)} - r_e \right) = \kappa_{di}^e \]
SFM-CAPM mark 2

Minimise:

\[ D^2 = D^2_\mu + hD^2_\kappa \]

where:

\[
D^2_\mu = \frac{1}{Q_\mu} \left[ \sum_{c=1}^{C} \left\{ \sum_{i=2}^{n_c} \left( \hat{\mu}_{ci}^{(S)} - \hat{\mu}_{ci}^{(G)} \right)^2 \right\} + \sum_{c=2}^{C} \left( \hat{\mu}_{c}^{(S)} - \hat{\mu}_{c}^{(G)} \right)^2 \right]
\]

\[
D^2_\kappa = \frac{1}{Q_\kappa} \sum_{c,e=1}^{C} \sum_{(d,i) \in \Psi_c} \left( \kappa_{di}^{c} - \kappa_{di}^{e} \right)^2
\]
## Data

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Expected returns to investors: nominal

Expected returns to investors:

- SFM-CAPM expected returns to investors
- GCAPM expected returns to investors

Distributions:
- zero D.mu
- low D.mu
- medium D.mu
- high D.mu
Expected returns to investors: real

SFM-CAPM expected returns to investors
GCAPM expected returns to investors: real returns

-0.05 0.00 0.05 0.10 0.15 0.20
-0.05 0.00 0.05 0.10 0.15 0.20

zero D.mu
low D.mu
medium D.mu
high D.mu
Optimal portfolios: home currency: nominal returns
Optimal portfolios: home currency: real returns

![Graph showing the relationship between pdic(S) and pdic(G) for different levels of D.mu.](image)

- **zero D.mu**
- **low D.mu**
- **medium D.mu**
- **high D.mu**
Betas: nominal returns
Betas: real returns

![Betas real returns graph]

- **bdic(S)**
- **bdic(G)**
- **zero D.mu**
- **low D.mu**
- **medium D.mu**
- **high D.mu**
\[
E \left\{ R_{di}^c \right\} = R_F^c + \beta_{di}^c \left[ E \left\{ R_M^c \right\} - R_F^c \right]
\]
Why adopt the SFM-CAPM?

• It’s better than the ICAPM.
• It’s better than the GCAPM.
• The difference is material.
A word of advice

Use real returns rather than nominal returns:

1) It’s closer to the GCAPM, so the adjustments required are smaller.

2) The CAPM is supposed to be about optimising consumption.

3) Financial mathematicians can’t use real returns; actuaries can.
Teşekkürler
Thank you